

INTERNATIONAL STANDARD



**Multicore and symmetrical pair/quad cables for digital communications –
Part 1-4: Assessment of conductor heating in bundled cables due to the
deployment of remote powering**

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INTERNATIONAL ELECTROTECHNICAL COMMISSION

**MULTICORE AND SYMMETRICAL PAIR/QUAD CABLES
FOR DIGITAL COMMUNICATIONS –****Part 1-4: Assessment of conductor heating in bundled cables
due to the deployment of remote powering**

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International Standard IEC 61156-1-4 has been prepared by subcommittee 46C: Wires and symmetric cables, of IEC technical committee 46: Cables, wires, waveguides, RF connectors, RF and microwave passive components and accessories.

This first edition cancels and replaces IEC PAS 61156-1-4, published in 2010. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition:

- a) Several test arrangements are considered;
- b) The mathematical approach was revised

The text of this document is based on the following documents:

CDV	Report on voting
46C/1089/CDV	46C/1098A/RVC

Full information on the voting for the approval of this International Standard can be found in the report on voting indicated in the above table.

This document has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts in the IEC 61156 series, published under the general title: *Multicore and symmetrical pair/quad cables for digital communications*, can be found on the IEC website.

The committee has decided that the contents of this document will remain unchanged until the stability date indicated on the IEC website under "<http://webstore.iec.ch>" in the data related to the specific document. At this date, the document will be

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- withdrawn,
- replaced by a revised edition, or
- amended.

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INTRODUCTION

The use of remote powering causes a temperature rise in cables installed in bundles.

This document specifies a method to assess the temperature increase in cable bundles by measuring the temperature change using thermocouples placed on the sheath of the cable at the centre of the bundle.

Several test arrangements are considered as well as current values.

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MULTICORE AND SYMMETRICAL PAIR/QUAD CABLES FOR DIGITAL COMMUNICATIONS –

Part 1-4: Assessment of conductor heating in bundled cables due to the deployment of remote powering

1 Scope

This document specifies a method to assess the thermal behaviour of cables arranged in bundles, for digital communications. A method is described to determine the temperature increase in cable bundles for certain remote powering currents as a basis to analyse the expected performance under several test arrangements described in standards dealing with the installation of cabling systems (e.g. [1]¹, [2]).

It is only intended to provide guidance to assess the thermal behaviour of cables for digital communications in reference environmental conditions and arrangements. It is therefore not intended to become a type of test method.

The following test arrangements are considered:

- c) free air;
- d) cable tray;
- e) conduits;
- f) conduit with sealed ends.

Even though this document covers four-pair data grade cables, the same principles can be applied to other cable types.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 61156-1, *Multicore and symmetrical pair/quad cables for digital communications – Part 1: Generic specification*

3 Terms, definitions and symbols

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in IEC 61156-1 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

¹ Numbers in square brackets refer to the Bibliography.

- IEC Electropedia: available at <http://www.electropedia.org/>
- ISO Online browsing platform: available at <http://www.iso.org/obp>

3.1.1

current carrying capacity ampacity

maximum amount of current that a conductor can carry while remaining within the maximum allowable temperature for the cable insulation

3.1.2

PoE

power over Ethernet

technology describing a system to pass electrical power safely, along with data, on Ethernet cabling

3.1.3

PoEP

power over Ethernet Plus

evolution of PoE to allow higher power

3.1.4

remote powering

supply of power to application-specific equipment via balanced cabling

3.1.5

temperature rise

difference in temperature (°C) between the initial temperature of the conductor surface without power and the final temperature at the surface of the powered conductor at steady state

3.2 Symbols

For the purposes of this document, the following symbols are used.

d	cable diameter (m)
i_c	current per conductor (A)
M	number of layers surrounding the centre cable
N	number of cables carrying remote powering current
n_c	number of conductors per cable carrying remote powering current i_c
R_{amb}	average DC resistance per unit length (Ω/m) of conductors carrying remote powering current at ambient temperature
$R(T)$	average DC resistance per unit length (Ω/m) of conductors carrying remote powering current at temperature T
T_{amb}	ambient temperature (°C)
T_c	temperature at the centre of the bundle (°C)
T_s	temperature of the outer surface of the bundle (°C)
ΔT	total temperature rise in reference to the ambient temperature
α	temperature coefficient of resistance
ρ_{th}	specific constant relating to cable construction
ρ_u	specific constant relating to environmental conditions
k_{th}	global constant relating to cable construction
k_u	global constant relating to environmental conditions

4 Testing procedure

4.1 General

In this document the current carrying capacity of conductors refers to the conductor surface temperature.

This temperature is of upmost importance in terms of the suitability of the insulation material at the interface with the conductor in order to avoid overheating and consecutive degradation of the insulation and transmission characteristics. In addition, health and safety issues may arise, such as a melt down and the potential danger of self-ignition.

The test protocol provides test configurations, methods and data submission formats that are necessary to produce effective comparative data.

4.2 Bundling of cables

To determine the maximum permissible heating of the data grade cables, consideration of the worst-case conditions prevailing is required. These conditions occur in tightly bundled cables.

Only cables with the densest hexagonal packing structure (see Figure 1) have a repetitive heat dissipation performance in a well-defined environment.

The densest hexagonal packing structure is characterized by the following number of cables in a bundle:

$$N = \left(1 + \sum_{i=1}^M 6 \times i \right) = 3 \times M^2 + 3 \times M + 1 \quad (1)$$

where

N is the total number of cables in the densest hexagonal packing structure
 $N = 1, 7, 19, 37, 61, 91, \dots$;

M is the number of layers surrounding the centre cable.

The quantities listed above are used in order to produce cable bundles with complete layers surrounding a centre cable as shown in Figure 1.

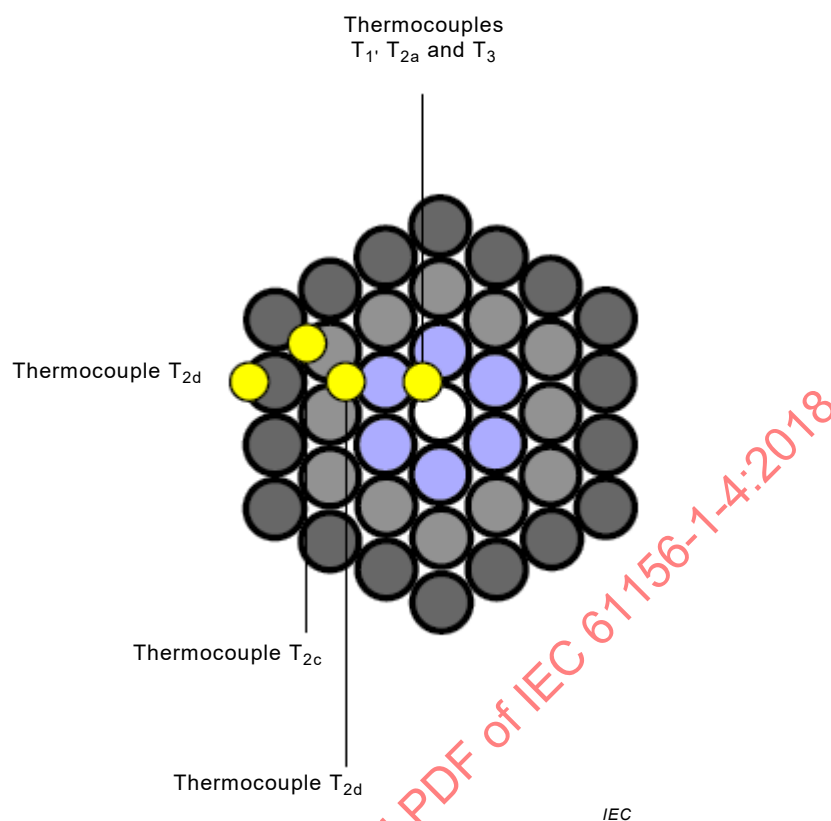


Figure 1 – Bundle of cables under test

NOTE Bundles can be constructed with lower numbers of cables (e.g. 7 or 19) where smaller containment systems are under investigation – but these are considered inappropriate for the needs of this document.

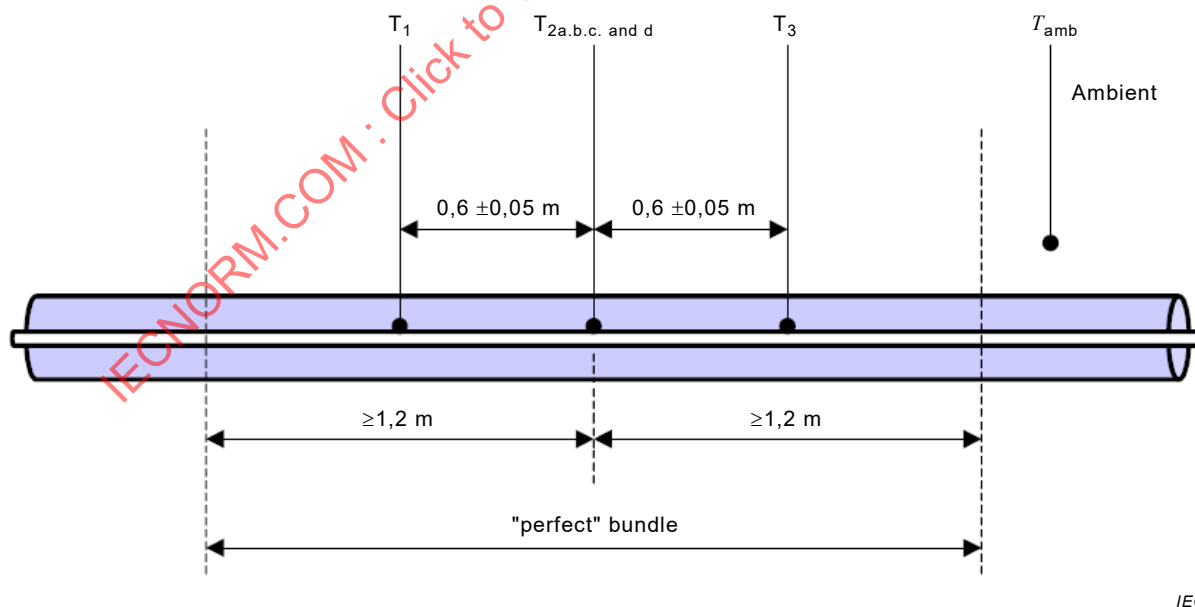


Figure 2 – Thermocouple placement

The bundle length should be about 4 m. To reduce end effects, the length of the centre cable shall be less than 80 % but more than 70 % of the length of the surrounding cable layers.

The cables are configured to allow the balanced pairs within them to be fed with a constant direct current. All conductors are connected in series, so the same current flows through the whole setup. It has been determined that only a eight-conductor powered configuration is required [3].

A minimum of six thermocouples, with size as small as possible to avoid the influence on temperature change, shall be placed within the length of the “perfect bundle”: four (T_{2a} , T_{2b} , T_{2c} and T_{2d}) shall be placed in the middle of the bundle length, with each one measuring the temperature of successive layers of cables, and one either side (T_1 and T_3) of the centre cable sheath at a distance of $(0,6 \pm 0,05)$ m from the middle. This configuration is shown in Figure 2.

A further thermocouple T_a is used to obtain ambient temperature information.

Additional thermocouples may be used to provide temperature information for particular arrangements (e.g. for fire barrier material placed around the central area of the bundle). Any information from these thermocouples is considered additional to that of T_1 , T_2 and T_3 .

NOTE The thermocouple configuration and bundle length described in this clause have been shown to provide a uniform temperature profile along the bundle. Shorter lengths show temperature drop-off towards the ends of the bundles.

4.3 Testing current and measurement accuracy

The tests are undertaken with eight conductors for each cable powered. The voltage shall be adjusted to maintain a stable current. The current stability shall be within 10 mA.

Thermocouples T_1 , T_2 and T_3 (and any other temperature profile thermocouples used) shall have an accuracy of ± 1 °C. The measurements obtained from T_1 , T_2 and T_3 shall be within ± 2 °C.

Thermocouple T_a may be a more appropriate ambient temperature meter (or built into a mass that limits sensitivity of the thermocouple to air movement).

4.4 Test conditions

In order to obtain comparative data, the following conductor current levels (i_c) should be used:

- a) 175 mA on four conductors acting as two pairs (equivalent to PoE);
- b) 300 mA on four conductors acting as two pairs (equivalent to PoEP);
- c) 300 mA on eight conductors, acting as four pairs;
- d) 500 mA on eight conductors, acting as four pairs.

Other currents may be used but the above conditions are chosen for the purposes of data comparison.

Applied current should remain for enough time to reach a stable final temperature.

4.5 Environmental conditions

It is intended that the temperature will rise when the cable bundle is subjected to a variety arrangements, including:

- a) free air;
- b) cable tray;
- c) conduit;
- d) conduit sealed at both ends;

and at different, higher ambient temperatures for all of the above.

Figures 3 to 6 are reported as an example of how to arrange a set-up for different environmental conditions [4].

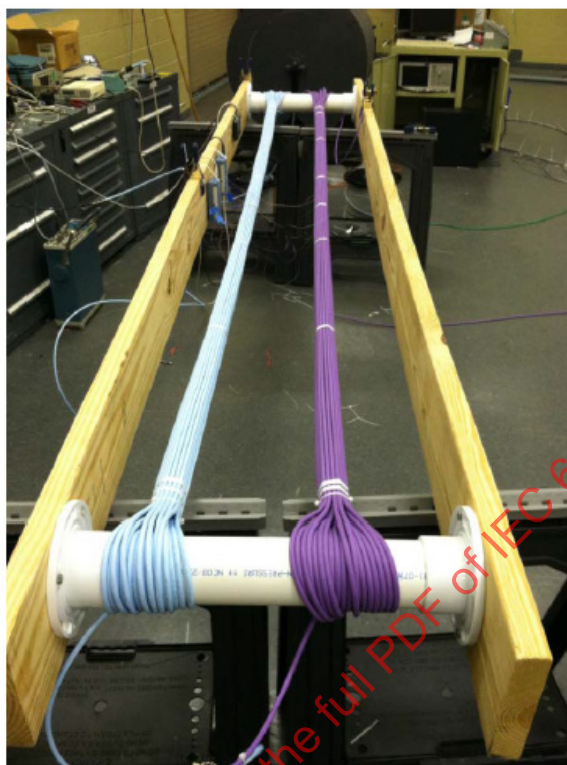
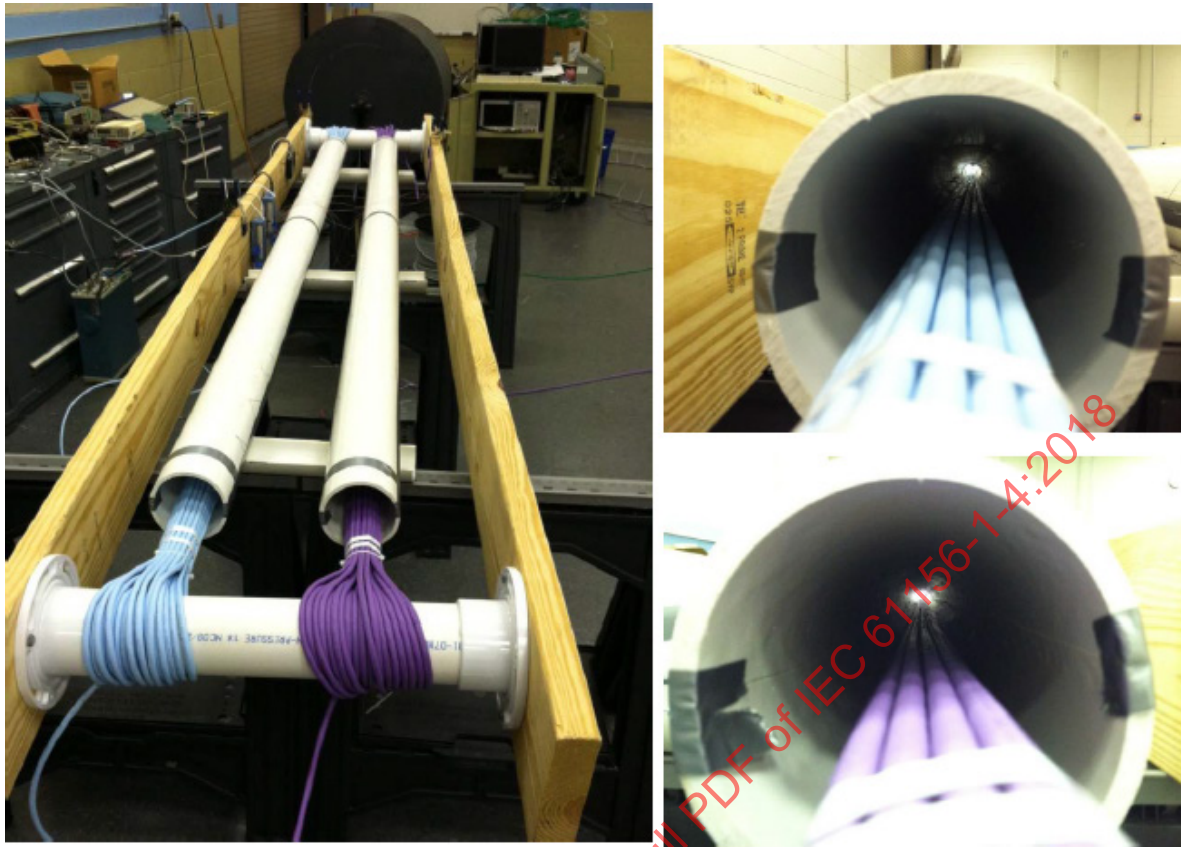


Figure 3 – Free air

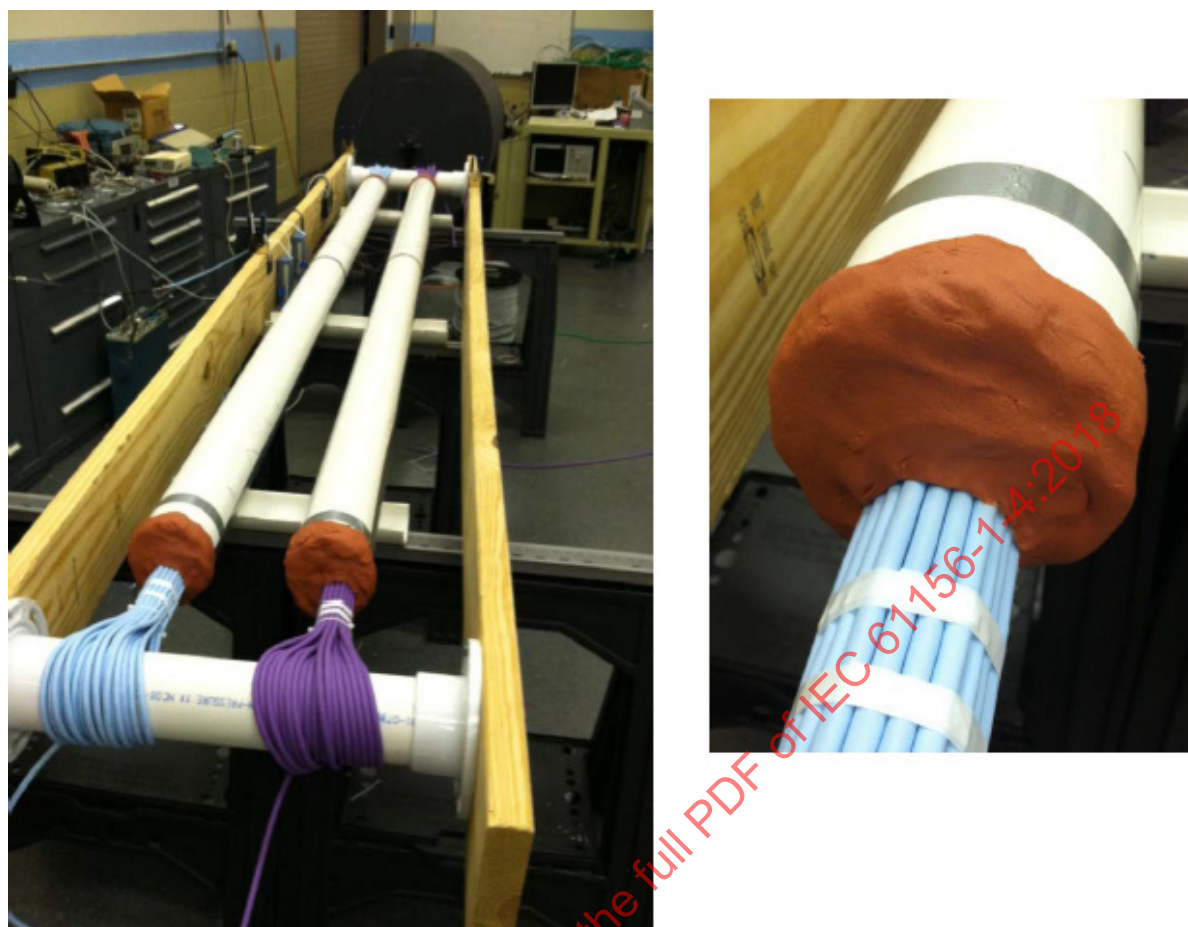


Figure 4 – Open conduit (Tray)



IEC

Figure 5 – Conduit



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Figure 6 – Sealed conduit

4.6 Assessment of the concatenated loop resistance of all pairs

To verify the suitability of the used constant current source, it is recommended to assess the total loop resistance of the bundle so as to have a minimum length suitable to match the used constant current source, the milli- or micro-voltmeter used and possibly a floating resistance meter or bridge.

4.7 Data submission

In order for the data to be directly compared, the following information shall be supplied:

- a) cable construction;
- b) cable diameter (data sheet information);
- c) cable bundle size (number of cables in the bundle);
- d) number of layers;
- e) the average resistance (Ω/m at $\sim 20^\circ\text{C}$) of the cable used – this may be obtained from a total resistance value of the rig divided by the actual physical length of the cable used;
- f) the average resistance (Ω/m) of the cable used at any other ambient temperature to be employed – this may be obtained from a total resistance value of the rig divided by the actual physical length of the cable used;
- g) the radial position of thermocouples T_{2a} , T_{2b} , T_{2c} and T_{2d} (if different than defined above);
- h) current injected (mA);
- i) duration of applied current before temperature reading;
- j) schematic showing placement of any additional thermocouples;

- k) a measurement of the relative humidity (at least at the point at which the cabling is judged to have reached steady state conditions);
- l) photographs of test environment and arrangement are recommended;
- m) test data over time:
 - 1) time stamp (recording start of test);
 - 2) voltage;
 - 3) current;
 - 4) temperature measured by thermocouple T_1 ;
 - 5) temperature measured by thermocouple T_{2a} ;
 - 6) temperature measured by thermocouple T_{3a} ;
 - 7) temperatures measured by additional thermocouples T_{2b} , T_{2c} and T_{2d} ;
 - 8) ambient temperature T_{amb} .

Table 1 and Table 2 show an example of recorded data:

Table 1 – Supporting information

Test file ID	001
Wires used	8
Cable type (U/UTP, etc.)	U/UTP
Cable diameter	7,3 mm
Bundle size (number of cables in the bundle)	37
Average conductor resistance	0,076 Ω /m at 21 °C (see note)
Environmental conditions	Free air
Relative humidity (at steady state heated condition)	41 %
Images attached	IMG001.JPG
NOTE Obtained from a DC loop resistance of 15,5 Ω for a physical length of 102 m.	

Table 2 – Test data format

Time min	T_1	T_{2a}	T_{2b}	T_{2c}	T_{2d}	T_3	T_{amb}	Voltage V	Current mA
0	21	21	21	21	21	21	21	24	300
5	22	22	22	21	21	22	21	24,2	300
10	23	23	23	22	22	23	21	24,3	300
15	25	25	24	24	23	25	21	24,4	300
20	27	27	26	25	24	27	21	24,5	300

4.8 Expression of results

The measurement results should be used to determine the coefficients of the interpolation formula to obtain the temperature rise as a function of bundle size for each current:

$$\Delta T(N, i_c) = (C_1 N + C_2 \sqrt{N}) \times i_c^2 \quad (2)$$

Details on Formula (2) are given in Annex A.

Annex A (informative)

Modelling of cable bundle heating [3]

A.1 Cable heating model

Empirical investigations have shown that in a cable bundle the temperature rise is a function of both current and number of cables.

The temperature rise of the cable at the centre of the bundle can be accurately approximated with the Function (A.1):

$$\Delta T(N, i_c) = (C_1 N + C_2 \sqrt{N}) \times i_c^2 \quad (^\circ\text{C}) \quad (\text{A.1})$$

where

ΔT is the total temperature rise at the centre of the cable bundle in reference to the ambient temperature;

N is the number of cables carrying remote powering current;

i_c is the current per conductor (A).

Coefficients C_1 and C_2 can be formulated as in Equations (A.2) and (A.3):

$$C_1 = \frac{\rho_{th} \times n_c}{4\pi} \times R(T) = k_{th} \times R(T) \quad (\text{A.2})$$

$$C_2 = \frac{\rho_u \times n_c}{d \times \left(\sqrt[4]{\frac{3}{4} \pi^6} \right)} \times R(T) = k_u \times R(T) \quad (\text{A.3})$$

As a first approximation, the average resistance of the conductors in the bundle can be expressed as in Equation (A.4) using for T the average of the centre (T_c) and surface temperature (T_s):

$$R(T) = R_{amb} \times \left[1 + \alpha \times \left(\frac{T_c + T_s}{2} - T_{amb} \right) \right] \quad (\text{A.4})$$

where

R_{amb} is the resistance at ambient temperature (Ω/m);

T_c is the temperature at the centre of the bundle ($^\circ\text{C}$);

T_s is the temperature at the outer surface of the bundle ($^\circ\text{C}$).

The total temperature rise at the centre of the cable bundle in reference to the ambient temperature can be expressed as in Equation (A.5):

$$\Delta T = \Delta T_{\text{bundle}} + \Delta T_{\text{surface}} = (T_c - T_s) + (T_s - T_{amb}) \quad (\text{A.5})$$

where $\Delta T_{\text{bundle}} \propto N$ and $\Delta T_{\text{surface}} \propto \sqrt{N}$.

The temperature difference between the centre and the surface can be written as in Equation (A.6):

$$\Delta T_{\text{bundle}} = T_c - T_s = \frac{\rho_{th} \times n_c}{4\pi} \times R(T) \times N \times i_c^2 = k_{th} \times R(T) \times N \times i_c^2 \quad (\text{A.6})$$

Equations (A.1), (A.4) and (A.6) can be written in matrix form, as in Formula (A.7):

$$\begin{bmatrix} 1 & \frac{-\alpha \times R_{amb}}{2} & \frac{-\alpha \times R_{amb}}{2} \\ -\left(k_{th} \times N + k_u \times \sqrt{N}\right) \times i_c^2 & 1 & 0 \\ k_{th} \times N \times i_c^2 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} R(T) \\ T_c \\ T_s \end{bmatrix} = \begin{bmatrix} R_{amb} \times (1 - \alpha \times T_{amb}) \\ T_{amb} \\ 0 \end{bmatrix} \quad (\text{A.7})$$

A more accurate approach takes into account the temperature of each layer in the bundle.

In the case of a 37-cable bundle, the conductor resistance might be better approximated by Equations (A.8) and (A.9):

$$R(T) = \frac{1}{37} \times (R_{\text{centre}} + 6 \times R_{\text{layer1}} + 12 \times R_{\text{layer2}} + 18 \times R_{\text{layer3}}) \quad (\text{A.8})$$

where

$$R_{\text{centre}} = R_{amb} \times [1 + \alpha \times (T_c - T_{amb})] \quad (\text{A.9})$$

If the temperatures T_1 , T_2 , T_3 of layers 1, 2, 3, ... were known, the terms in Equation (A.8) could be written as Equations (A.10), (A.11) and (A.12):

$$R_{\text{layer1}} = R_{amb} \times [1 + \alpha \times (T_1 - T_{amb})] \quad (\text{A.10})$$

$$R_{\text{layer2}} = R_{amb} \times [1 + \alpha \times (T_2 - T_{amb})] \quad (\text{A.11})$$

$$R_{\text{layer3}} = R_{amb} \times [1 + \alpha \times (T_3 - T_{amb})] \quad (\text{A.12})$$

The temperatures T_1 , T_2 , T_3 can be approximated as linear functions of the temperatures T_c and T_s . When considering the resistance of the m^{th} layer of a bundle containing M layers surrounding a centre cable, the temperature T_m is as illustrated in Equation (A.13):

$$T_m \cong T_c - \frac{2 \cdot m}{2 \cdot M + 1} \times (T_c - T_s) \quad (\text{A.13})$$

In the case of a bundle of 37 cables, Equations (A.10) to (A.12) become:

$$R_{layer1} \cong R_{amb} \times \left[1 + \alpha \times \left(T_c - \frac{2}{7} \cdot (T_c - T_s) - T_{amb} \right) \right] \quad (A.14)$$

$$R_{layer2} \cong R_{amb} \times \left[1 + \alpha \times \left(T_c - \frac{4}{7} \cdot (T_c - T_s) - T_{amb} \right) \right] \quad (A.15)$$

$$R_{layer3} \cong R_{amb} \times \left[1 + \alpha \times \left(T_c - \frac{6}{7} \cdot (T_c - T_s) - T_{amb} \right) \right] \quad (A.16)$$

This more accurate version of the resistance for the case of a 37-cable bundle changes Equation (A.7) as shown in Equation (A.17):

$$\begin{bmatrix} 1 & \frac{-13 \cdot \alpha \times R_{amb}}{37} & \frac{-24 \cdot \alpha \times R_{amb}}{37} \\ -\left(k_{th} \times N + k_u \times \sqrt{N}\right) \times i_c^2 & 1 & 0 \\ k_{th} \times N \times i_c^2 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} R(T) \\ T_c \\ T_s \end{bmatrix} = \begin{bmatrix} R_{amb} \times (1 - \alpha \times T_{amb}) \\ T_{amb} \\ 0 \end{bmatrix} \quad (A.17)$$

The case of a 37-cable bundle can be generalized to any cable bundles containing M layers surrounding a centre cable, in which $R(T)$ becomes as given in Equation (A.18):

$$R(T) = \frac{R_{amb}}{M^2 + M - 1} \times \left[\left(M^2 + M + 1 \right) \times \left(\alpha \times T_c - 3 \cdot \alpha \times T_{amb} + 2 \cdot \alpha \times T_s + 3 \right) - 2 \cdot \alpha \times T_s \right] \quad (A.18)$$

And the solution to Equation (A.17) becomes as in Equation (A.19):

$$\begin{bmatrix} R(T) \\ T_c \\ T_s \end{bmatrix} = \begin{bmatrix} 1 & \frac{\left(M^2 + M + 1 \right) \times \alpha \times R_{amb}}{\left(3 \times M^2 + 3 \times M + 1 \right)} & \frac{2 \times \left(M^2 + M \right) \times \alpha \times R_{amb}}{\left(3 \times M^2 + 3 \times M + 1 \right)} \\ -\left(k_{th} \times N + k_u \times \sqrt{N}\right) \times i_c^2 & 1 & 0 \\ k_{th} \times N \times i_c^2 & -1 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} R_{amb} \times (1 - \alpha \times T_{amb}) \\ T_{amb} \\ 0 \end{bmatrix} \quad (A.19)$$