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TECHNICAL REPORT

Electromagnetic compatibility (EMC) –
Part 1-7: General – Power factor in single-phase systems under non-sinusoidal conditions

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ELECTROMAGNETIC COMPATIBILITY (EMC) –

Part 1-7: General – Power factor in single-phase systems under non-sinusoidal conditions

FOREWORD

The main task of IEC technical committees is to prepare International Standards. However, a technical committee may propose the publication of a technical report when it has collected data of a different kind from that which is normally published as an International Standard, for example "state of the art".

IEC TR 61000-1-7, which is a Technical Report, has been prepared by subcommittee 77A: *EMC – Low frequency phenomena,* of IEC technical committee 77*: Electromagnetic compatibility*.

The text of this technical report is based on the following documents:

Enquiry draft	Report on voting
77A/911/DTR	77A/920/RVC

Full information on the voting for the approval of this technical report can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

A list of all parts in the IEC 61000 series, published under the general title *Electromagnetic compatibility (EMC)*, can be found on the IEC website.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC website under "http://webstore.iec.ch" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

A bilingual version of this publication may be issued at a later date.

INTRODUCTION

0.1 Series overview

IEC 61000 is published in separate parts, according to the following structure:

Part 1: General

General considerations (introduction, fundamental principles)

Definitions, terminology

Part 2: Environment

Description levels

Classification of the environment

Compatibility levels

Part 3: Limits

Emission limits

Immunity limits (in so far as they do not fall under the responsibility of the product committees)

Part 4: Testing and measurement techniques

Measurement techniques

Testing techniques

Part 5: Installation and mitigation guidelines

Installation guidelines

Mitigation methods and devices

Part 6: Generic standards

Part 9: Miscellaneous

Each part is further subdivided into sections which are to be published either as international standards, technical specifications, or as technical reports.

These standards and reports will be published in chronological order and numbered accordingly (for example, 61000-6-1).

0.2 Purpose of this document

The prevalence of loads drawing non-sinusoidal current from power systems requires clarification of such concepts as power and power factor, in order to avoid confusion due to implied assumptions of sinusoidal voltage and current. This document specifically addresses the terms related to the power factor of equipment that are applicable regardless of the voltage and current waveforms.

When voltages and currents on power supply networks are perfectly sinusoidal, cos φ corresponds to the power factor. But this is not true anymore when electric quantities are distorted. In some existing documents, cos φ is still used as power factor, leading to an incorrect assessment of the equipment impact to supply networks.

The purpose of this Technical Report is to give clear information on both components in the power factor:

- the fundamental power factor, which is due to the phase difference between the voltage and current at the fundamental frequency (cos φ_1), and
- the non-fundamental power factor, which is related to the distortion of the voltage and/or current.

ELECTROMAGNETIC COMPATIBILITY (EMC) –

Part 1-7: General – Power factor in single-phase systems under non-sinusoidal conditions

1 Scope

This part of IEC 61000, which is a Technical Report, provides definitions of various electrical power quantities and the relationship between them under non-sinusoidal conditions, in order to give clear information on both components in the power factor: the fundamental power factor, which is due to the phase difference between the voltage and current at the fundamental frequency, and the non-fundamental power factor, which is related to the distortion of the voltage and/or current. This Technical Report is applicable only to single-phase systems.

This Technical Report provides definitions for the three following cases:

- the general case where the voltage and current are both distorted (Clause 5),
- the case where the voltage is assumed to be sinusoidal and the current is only distorted with harmonic components (Clause 6),
- the particular case where the voltage and current are both sinusoidal (Annex A).

Annex B gives information on the fundamental active factor, which is used to describe the behaviour of a piece of equipment as a load or a generator.

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

Void.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1
root-mean-square value
r.m.s. value
effective value
for a time-dependent quantity, positive square root of the mean value of the square of the quantity taken over a given time interval

Note 1 to entry: The root-mean-square value of a periodic quantity is usually taken over an integration interval the range of which is the period multiplied by a natural number.

Note 2 to entry: For a sinusoidal quantity $a(t) = \hat{A}\cos(\omega t + \vartheta_0)$, the root-mean-square value is $A_{\mathrm{eff}} = \hat{A}/\sqrt{2}$.

Note 3 to entry: The root-mean-square value of a quantity may be denoted by adding one of the subscripts eff or rms to the symbol of the quantity.

Note 4 to entry: In electrical technology, the root-mean-square values of electric current $i(t)$ and voltage $u(t)$are usually denoted I and U, respectively.

[SOURCE: IEC 60050-103:2009, 103-02-03]

3.2
direct component
mean value of a quantity taken over a given time interval

[SOURCE: IEC 60050-103:2009, 103-06-05, modified – definition extended to quantities containing interharmonic components.]

3.3
sinusoidal, adj.
pertaining to an alternating quantity represented by the product of a real constant and a sine or cosine function whose argument is a linear function of the independent variable

Note 1 to entry: The real constant may be a scalar, vector or tensor quantity.

Note 2 to entry: Examples are $a(t) = \hat{A}\cos(\omega t + \vartheta_0)$ and $a(x) = \hat{A}\cos[k(x - x_0)]$.

[SOURCE: IEC 60050-103:2009, 103-07-01]

3.4
initial phase
phase angle
ϑ_0
value of the phase of a sinusoidal quantity when the value of the independent variable is zero

Note 1 to entry: For the quantity $a(t) = \hat{A}\cos(\omega t + \vartheta_0)$, the initial phase is ϑ_0.

[SOURCE: IEC 60050-103:2009, 103-07-05]

3.5
periodic conditions
state of an electric circuit element or electric circuit that is characterized by the electric currents and voltages all being periodic functions of time with the same period T

[SOURCE: IEC 60050-131:2002, 131-11-27, modified – addition of symbol T for the period.]

3.6
sinusoidal conditions
state of a linear electric circuit element or electric circuit that is characterized by the electric currents and voltages all being sinusoidal functions of time with the same frequency

[SOURCE: IEC 60050-131:2002, 131-11-28]

3.7
instantaneous power
$p(t)$
for a two-terminal element or a two-terminal circuit with terminals A and B, product of the voltage u_{AB} between the terminals and the electric current i in the element or circuit

$$p(t) = u_{AB}(t) \cdot i(t)$$

where u_{AB} is the line integral of the electric field strength from A to B, and where the electric current in the element or circuit is taken positive if its direction is from A to B and negative if its direction is from B to A

Note 1 to entry: The direction of electric current is as defined in IEC 60050:2002, 131-11-29.

Note 2 to entry: In circuit theory the electric field strength is generally non-rotational and thus $u_{AB} = v_A - v_B$, where v_A and v_B are the electric potentials at terminals A and B, respectively.

Note 3 to entry: The coherent SI unit of instantaneous power is watt, W.

Note 4 to entry: A two-terminal element or circuit refers to a single-phase equipment or system.

[SOURCE: IEC 60050-131:2013, 131-11-30, modified – in note 2, the term irrotational is replaced by non-rotational and a new note 4 has been added.]

3.8
apparent power
S
product of the r.m.s. voltage U between the terminals of a two-terminal element or two-terminal circuit and the r.m.s. electric current I in the element or circuit

$$S = UI$$

Note 1 to entry: The coherent SI unit for apparent power is voltampere, VA.

Note 2 to entry: A two-terminal element or circuit refers to a single-phase equipment or system.

[SOURCE: IEC 60050-131:2013, 131-11-41, modified – the existing note 1 has been removed and a note 2 has been added.]

3.9
active power
P
under periodic conditions, mean value, taken over one period T, of the instantaneous power $p(t)$

$$P = \frac{1}{T}\int_0^T p(t)dt$$

Note 1 to entry: The coherent SI unit for active power is watt, W.

Note 2 to entry: When the voltage or current contain interharmonic components, often their waveforms are no more periodic. In this document, the active power is approximated by the mean value of the instantaneous power, taken over an integer number of periods of the a.c. power supply system (see 5.3.1 and 5.1.4). This definition is also used under periodic conditions in this document (see 6.3 and Clause A.3).

[SOURCE: IEC 60050-131:2013, 131-11-42, modified – the existing note 1 has been removed and a note 2 has been added.]

3.10
non-active power
$Q\sim$
for a two-terminal element or a two-terminal circuit under periodic conditions, quantity equal to the square root of the difference of the squares of the apparent power S and the active power P

$$Q\sim = \sqrt{S^2 - P^2}$$

Note 1 to entry: The coherent SI unit for non-active power is voltampere, VA. The special name "var" and its symbol "var" are also used. See IEC 60050-131:2013, 131-11-45.

Note 2 to entry: A two-terminal element or circuit refers to a single-phase equipment or system.

[SOURCE: IEC 60050-131:2013, 131-11-43, modified – the existing note 1 has been removed and a note 2 has been added.]

3.11
reactive power
Q
for a linear two-terminal element or two-terminal circuit, under sinusoidal conditions, quantity equal to the product of the apparent power S and the sine of the displacement angle φ

$$Q = S\sin\varphi$$

Note 1 to entry: The coherent SI unit for reactive power is voltampere, VA. The special name var and its symbol var are also used. See IEC 60050-131:2013, 131-11-45.

Note 2 to entry: A two-terminal element or circuit refers to a single-phase equipment or system.

Note 3 to entry: When the conditions are not sinusoidal, there is no international consensus on a definition of the reactive power. Instead, several definitions of the reactive power exist. In some documents, the reactive power is taken as the non-active power, but there are many other formulae.

[SOURCE: IEC 60050-131:2013, 131-11-44, modified – the existing note 1 has been removed and notes 2 and 3 have been added.]

3.12
power factor
λ
ratio of the absolute value of the active power P to the apparent power S

$$\lambda = \frac{|P|}{S}$$

Note 1 to entry: Under sinusoidal conditions, the power factor is the absolute value of the active factor.

[SOURCE: IEC 60050-131:2002, 131-11-46, modified – definition extended to quantities containing interharmonic components.]

3.13
displacement angle
phase difference angle
φ
under sinusoidal conditions, phase difference between the voltage applied to a linear two-terminal element or two-terminal circuit and the electric current in the element or circuit

Note 1 to entry: The cosine of the displacement angle is the active factor.

Note 2 to entry: A two-terminal element or circuit refers to a single-phase equipment or system.

[SOURCE: IEC 60050-131:2002, 131-11-48, modified – a note 2 has added.]

3.14
active factor
for a two-terminal element or a two-terminal circuit under sinusoidal conditions, ratio of the active power to the apparent power

Note 1 to entry: The active factor is equal to the cosine of the displacement angle, and can vary from –1 to +1.

Note 2 to entry: A two-terminal element or circuit refers to a single-phase equipment or system.

[SOURCE: IEC 60050-131:2002, 131-11-49, modified – a note 2 has been added.]

3.15
fundamental component
fundamental
sinusoidal component of the Fourier series of a periodic quantity having the frequency of the quantity itself

[SOURCE: IEC 60050-103:2009, 103-07-19]

3.16
reference fundamental component
conventionally chosen sinusoidal component of a quantity, to the frequency of which all the other components are referred

Note 1 to entry: The term is used when, for a periodic quantity, the chosen component differs from the fundamental component, or when the quantity is not periodic due to interharmonic components.

Note 2 to entry: In this document, the component having the frequency of the a.c. supply system is chosen as the reference fundamental component.

[SOURCE: IEC 60050-103:2009, 103-07-20, modified – definition extended to quantities containing interharmonic components and a note 2 added.]

3.17
fundamental frequency
frequency of the fundamental component of a periodic quantity

[SOURCE: IEC 60050-103:2009, 103-07-21]

3.18
reference fundamental frequency
frequency of the reference fundamental component

Note 1 to entry: The term is used when, for a periodic quantity, the reference fundamental component differs from the fundamental component, or when the quantity is not periodic due to interharmonic components.

[SOURCE: IEC 60050-103:2009, 103-07-22, modified – definition extended to quantities containing interharmonic components.]

3.19
harmonic frequency
frequency which is an integer multiple greater than one of the fundamental frequency or of the reference fundamental frequency

Note 1 to entry: When a reference fundamental frequency is defined, it is used in place of the fundamental frequency.

Note 2 to entry: In this document, the harmonic frequencies are always related to the frequency of the a.c. power supply system.

[SOURCE: IEC 60050-551:2001, 551-20-05, modified – addition of notes 1 and 2.]

3.20
interharmonic frequency
frequency which is a non-integer multiple of the reference fundamental frequency

[SOURCE: IEC 60050-551:2001, 551-20-06]

3.21
harmonic component
sinusoidal component of a quantity having a harmonic frequency

[SOURCE: IEC 60050-551:2001, 551-20-07, modified – definition extended to quantities containing interharmonic components.]

3.22
interharmonic component
sinusoidal component of a quantity having an interharmonic frequency

[SOURCE: IEC 60050-551:2001, 551-20-08, modified – definition extended to quantities containing interharmonic components.]

3.23
harmonic order
ratio of the frequency of any sinusoidal component to the fundamental frequency or the reference fundamental frequency

Note 1 to entry: The harmonic order of the fundamental component or the reference fundamental component is one.

Note 2 to entry: When a reference fundamental frequency is defined, it is used in place of the fundamental frequency.

[SOURCE: IEC 60050-551:2001, 551-20-09, modified – addition of note 2.]

3.24
total distortion content
quantity obtained by subtracting from a quantity its direct component and its fundamental component or its reference fundamental component

Note 1 to entry: The total distortion content includes harmonic components and interharmonic components if any.

Note 2 to entry: When a reference fundamental frequency is defined, the reference fundamental component is used in place of the fundamental component.

Note 3 to entry: The total distortion content is a time function.

[SOURCE: IEC 60050-551:2001, 551-20-11, modified – definition extended to quantities containing interharmonic components and note 4 deleted.]

3.25
harmonic content
sum of the harmonic components of a quantity

Note 1 to entry: The harmonic content is a time function.

[SOURCE: IEC 60050-551:2001, 551-20-12, modified – definition extended to quantities containing interharmonic components and notes 2 and 3 deleted.]

3.26
total harmonic ratio
total harmonic distortion
THD
ratio of the r.m.s. value of the harmonic content to the r.m.s. value of the fundamental component or the reference fundamental component of a quantity

Note 1 to entry: When a reference fundamental frequency is defined, the reference fundamental component is used in place of the fundamental component.

Note 2 to entry: The total harmonic ratio can be approximated by limiting the calculation up to a certain harmonic order.

[SOURCE: IEC 60050-551:2001, 551-20-13, modified – definition extended to quantities containing interharmonic components.]

3.27
total distortion ratio
ratio of the r.m.s. value of the total distortion content to the r.m.s. value of the fundamental component or the reference fundamental component of a quantity

Note 1 to entry: When a reference fundamental frequency is defined, the reference fundamental component is used in place of the fundamental component.

Note 2 to entry: The total distortion ratio can be approximated by limiting the calculation up to a certain harmonic order.

[SOURCE: IEC 60050-551:2001, 551-20-14, modified – definition extended to quantities containing interharmonic components.]

3.28
fundamental factor
ratio of the r.m.s. value of the fundamental component or the reference fundamental component to the r.m.s. value of a quantity

Note 1 to entry: When a reference fundamental frequency is defined, the reference fundamental component is used in place of the fundamental component.

[SOURCE: IEC 60050-551:2001, 551-20-17, modified – definition extended to quantities containing interharmonic components.]

4 General

This Technical Report provides definitions of various electrical power quantities and the relationship between them, when a voltage $u(t)$ is applied to a single-phase equipment or system, $i(t)$ being the current flowing in the equipment or system.

The following cases are considered:

- In Clause 5, the general case is described. The voltage and current are both distorted and can contain d.c., harmonic and interharmonic components.
- In Clause 6, the voltage is assumed to be sinusoidal and the current is only distorted with harmonic components.
- In Annex A, the voltage and current are both sinusoidal.

In this document, the reference fundamental frequency is the frequency of the a.c. supply power system (assumed to be constant, but not necessarily equal to the rated value of 50 Hz or 60 Hz) and all harmonic or interharmonic frequencies are related to this frequency.

In the particular case where the voltage is sinusoidal and the current does not contain interharmonic components, these quantities are periodic and their fundamental frequency is equal to the frequency of the a.c. power supply system. Therefore, the term "reference fundamental frequency" is replaced by the term "fundamental frequency" in Clause 6 and Annex A.

In this Technical Report, the harmonic order of harmonic or interharmonic components is not limited. But, in practical applications, the harmonic order may be limited to a specified order h_{max}. Usually, h_{max} is equal to 40 or 50.

NOTE h_{max} can be as low as 15 for performance measurement and monitoring devices (PMDs) defined by IEC 61557-12.

5 Electric power quantities under non-sinusoidal conditions

5.1 Voltages and currents

5.1.1 Instantaneous values

For steady-state conditions, the non-sinusoidal instantaneous value of the voltage or current is the sum of a d.c. component, a sinusoidal component at the power system frequency and sinusoidal components at harmonic or interharmonic frequencies.

$$u(t) = U_0 + U_1\sqrt{2}\cdot\sin(\omega t + \alpha_1) + \sum_{\substack{m>0\\ m\neq 1}} U_m\sqrt{2}\cdot\sin(m\omega t + \alpha_m)$$

$$i(t) = I_0 + I_1\sqrt{2}\cdot\sin(\omega t + \beta_1) + \sum_{\substack{m>0\\ m\neq 1}} I_m\sqrt{2}\cdot\sin(m\omega t + \beta_m)$$

where

$u(t)$ is the instantaneous value of the voltage at time t;

U_0 is the d.c. component of the voltage;

U_1 is the r.m.s. value of the reference fundamental component of the voltage;

ω is the angular frequency corresponding to the reference fundamental frequency;

t is the time;

α_1 is the initial phase of the reference fundamental component of the voltage;

m is the harmonic order (m is a positive real number different from 0 and 1. It is an integer number for harmonic components and a non-integer number for interharmonic components);

U_m is the r.m.s. value of the voltage harmonic or interharmonic component of order m;

α_m is the initial phase of the voltage harmonic or interharmonic component of order m;

$i(t)$ is the instantaneous value of the current at time t;

I_0 is the d.c. component of the current;

I_1 is the r.m.s. value of the reference fundamental component of the current;

β_1 is the initial phase of the reference fundamental component of the current;

I_m is the r.m.s. value of the current harmonic or interharmonic component of order m;

β_m is the initial phase of the current harmonic or interharmonic component of order m.

The non-sinusoidal instantaneous value of the voltage or current can also be written as the sum of three terms:

$$u(t) = U_0 + u_1(t) + u_\mathrm{D}(t)$$

$$i(t) = I_0 + i_1(t) + i_\mathrm{D}(t)$$

where

$u_1(t)$ is the reference fundamental component of the voltage at time t (see 5.1.2);

$u_\mathrm{D}(t)$ is the total distortion content of the voltage at time t (see 5.1.3);

$i_1(t)$ is the reference fundamental component of the current at time t (see 5.1.2);

$i_\mathrm{D}(t)$ is the total distortion content of the current at time t (see 5.1.3).

5.1.2 Reference fundamental components

The reference fundamental component of the voltage (current) is defined as the sinusoidal component of the voltage (current) having the frequency of the a.c. power supply system:

$$u_1(t) = U_1\sqrt{2} \cdot \sin(\omega t + \alpha_1)$$

$$i_1(t) = I_1\sqrt{2} \cdot \sin(\omega t + \beta_1)$$

5.1.3 Total distortion contents

The total distortion content of the voltage (current) is obtained by subtracting the direct component and the reference fundamental component from the instantaneous value of the voltage (current):

$$u_D(t) = u(t) - U_0 - u_1(t)$$

$$i_D(t) = i(t) - I_0 - i_1(t)$$

Therefore, according to 5.1.1:

$$u_D(t) = \sum_{\substack{m>0 \\ m\neq 1}} U_m\sqrt{2} \cdot \sin(m\omega t + \alpha_m)$$

$$i_D(t) = \sum_{\substack{m>0 \\ m\neq 1}} I_m\sqrt{2} \cdot \sin(m\omega t + \beta_m)$$

5.1.4 RMS values of the voltage and current

In this document, the r.m.s. value of the voltage U (current I) is defined as the positive square root of the mean value of the square of the voltage (current) taken over an integer number of periods of the a.c. power supply system:

$$U = \sqrt{\frac{1}{kT}\int_{\tau}^{\tau+kT}[u(t)]^2\,dt}$$

$$I = \sqrt{\frac{1}{kT}\int_{\tau}^{\tau+kT}[i(t)]^2\,dt}$$

where

T is the reciprocal of the reference fundamental frequency;

k is an integer number;

τ is the time when the measurement starts.

NOTE If the voltage and current are only distorted with harmonic components, then a measurement time interval kT enables the correct measurement of r.m.s. and power values. If the voltage or the current contain interharmonic components, r.m.s. and power values are incorrectly measured, unless the measurement time interval kT includes an integer number of the period of each interharmonic component. For practical situations when the bulk of the power is carried by the reference fundamental components, such errors are small. The larger the measurement time interval kT, the less significant the errors caused by interharmonic components become. For more information, see IEEE Std 1459-2010. Practical methods to evaluate signals that contain interharmonics or fluctuating components are presented in the future IEC TR 61000-4-40.

These quantities are the sum of several terms:

$$U^2 = U_0^2 + U_1^2 + U_D^2$$

$$I^2 = I_0^2 + I_1^2 + I_D^2$$

where

U_0 is the d.c. component of the voltage;

U_1 is the r.m.s. value of the reference fundamental component of the voltage;

U_D is the r.m.s. value of the total distortion content of the voltage (see 5.1.5);

I_0 is the d.c. component of the current;

I_1 is the r.m.s. value of the reference fundamental component of the current;

I_D is the r.m.s. value of the total distortion content of the current (see 5.1.5).

5.1.5 RMS values of total distortion contents

According to the definition given in 5.1.3, the r.m.s. value of the total distortion content of the voltage (current) is defined as follows:

$$U_D^2 = U^2 - U_0^2 - U_1^2$$

$$I_D^2 = I^2 - I_0^2 - I_1^2$$

According to 5.1.3, these quantities are also given by:

$$U_D^2 = \sum_{\substack{m>0 \\ m\neq 1}} U_m^2$$

$$I_D^2 = \sum_{\substack{m>0 \\ m\neq 1}} I_m^2$$

5.1.6 DC ratios

The d.c. ratio of the voltage DCR_U (current DCR_I) is defined as the ratio of its d.c. component to the r.m.s. value of its reference fundamental component:

$$DCR_U = \frac{U_0}{U_1}$$

$$DCR_I = \frac{I_0}{I_1}$$

5.1.7 Total distortion ratios

The total distortion ratio of the voltage TDR_U (current TDR_I) is defined as the ratio of the r.m.s. value of its total distortion content to the r.m.s. value of its reference fundamental component:

$$TDR_U = \frac{U_D}{U_1}$$

$$TDR_I = \frac{I_D}{I_1}$$

NOTE If there is no interharmonic component, the total distortion ratio is equal to the total harmonic ratio *THD* (also called total harmonic distortion). See 6.1.6.

5.2 Instantaneous power

According to definition 3.7, the instantaneous power $p(t)$ is defined as the product of the voltage $u(t)$ and the electric current $i(t)$:

$$p(t) = u(t) \cdot i(t)$$

5.3 Definitions related to the active power

5.3.1 Active power

When the voltage or current contain interharmonic components, often their waveforms are no longer periodic. In that case, an approximation of the active power P is the mean value, taken over an integer number of periods kT, of the instantaneous power $p(t)$:

$$P = \frac{1}{kT} \int_{\tau}^{\tau+kT} p(t)\, dt$$

NOTE See also the note in 5.1.4.

The active power is the sum of several components:

$$P = U_0 I_0 + U_1 I_1 \cos\varphi_1 + \sum_{\substack{m>0 \\ m\neq 1}} U_m I_m \cos\varphi_m$$

where

φ_m is the phase angle between the voltage and current components of order m.

$$\varphi_m = \alpha_m - \beta_m$$

The active power can also be written as the sum of three terms:

$$P = P_0 + P_1 + P_D$$

where

P_0 is the d.c. power (see 5.3.2);

P_1 is the fundamental active power (see 5.3.3);

P_D is the distortion active power (including harmonic and interharmonic power) (see 5.3.4).

5.3.2 DC power

The d.c. power P_0 is defined as the product of the direct component of the voltage and the direct component of the current:

$$P_0 = U_0 I_0$$

5.3.3 Fundamental active power

The fundamental active power P_1 is defined as the mean value, taken over an integer number of periods kT, of the product of the reference fundamental components of the voltage $u_1(t)$ and the current $i_1(t)$:

$$P_1 = \frac{1}{kT}\int_{\tau}^{\tau+kT}\left[u_1(t)\cdot i_1(t)\right]dt$$

The fundamental active power is also given by:

$$P_1 = U_1 I_1 \cos\varphi_1$$

5.3.4 Distortion active power

The distortion active power P_D is obtained by subtracting the d.c. power P_0 and the fundamental active power P_1 from the active power P:

$$P_D = P - P_0 - P_1$$

According to 5.3.1, the distortion active power is also given by:

$$P_D = \sum_{\substack{m>0 \\ m\neq 1}} U_m I_m \cos\varphi_m$$

5.4 Definitions related to the apparent power

5.4.1 Apparent power

The apparent power S is defined as the product of the r.m.s. value of the voltage U and the r.m.s. value of the electric current I:

$$S = U \cdot I$$

NOTE The apparent power corresponds to the maximum active power that can be transmitted to a load or system through a given line, while keeping the r.m.s. value of the voltage constant and the line losses constant. This maximum value of active power is obtained when the load or system is supplied with in phase sinusoidal voltage and current.

Using the separation of the r.m.s. voltage and current into their main components (see 5.1.4), the apparent power can also be written as follows:

$$S^2 = \left(U_0^2 + U_1^2 + U_D^2\right)\times\left(I_0^2 + I_1^2 + I_D^2\right)$$

$$S^2 = (U_1 I_1)^2 \times\left[1 + (U_0/U_1)^2 + (U_D/U_1)^2\right]\times\left[1 + (I_0/I_1)^2 + (I_D/I_1)^2\right]$$

$$S = S_1 \times \sqrt{1 + DCR_U^2 + TDR_U^2} \times \sqrt{1 + DCR_I^2 + TDR_I^2}$$

where

S is the apparent power;

S_1 is the fundamental apparent power (see 5.4.2);

DCR_U is the d.c. ratio of the voltage;

TDR_U is the total distortion ratio of the voltage;

DCR_I is the d.c. ratio of the current;

TDR_I is the total distortion ratio of the current.

5.4.2 Fundamental apparent power

The fundamental apparent power S_1 is defined as the product of the r.m.s. values of the reference fundamental component of the voltage U_1 and the reference fundamental component of the current I_1:

$$S_1 = U_1 I_1$$

As for the particular case where the voltage and current are both sinusoidal, the fundamental apparent power has two components:

$$S_1^2 = P_1^2 + Q_1^2$$

where

P_1 is the fundamental active power;

Q_1 is the fundamental reactive power.

$$P_1 = U_1 I_1 \cos \varphi_1$$

$$Q_1 = U_1 I_1 \sin \varphi_1$$

5.5 Definitions related to the power factor

5.5.1 Power factor

As under periodic conditions, the power factor λ is defined as the ratio of the absolute value of the active power P to the apparent power S:

$$\lambda = \frac{|P|}{S}$$

NOTE Since the apparent power is the maximum active power that can be transmitted to a load or system through a given line (see 5.4.1), the power factor is a utilization factor indicator.

Using the separation of the active power and apparent power into their main components (see 5.3.1 and 5.4.1), the power factor can be written as follows:

$$\lambda = \frac{|P_1| \times |1 + (P_0 / P_1) + (P_D / P_1)|}{S_1 \times \sqrt{1 + DCR_U^2 + TDR_U^2} \times \sqrt{1 + DCR_I^2 + TDR_I^2}}$$

So, the power factor can be written as the product of two factors:

$$\lambda = \lambda_1 \cdot \lambda_N$$

where

λ_1 is the fundamental power factor (see 5.5.2);

λ_N is the non-fundamental power factor (see 5.5.3).

5.5.2 Fundamental power factor

The fundamental power factor λ_1 is defined as the ratio of the absolute value of the fundamental active power P_1 to the fundamental apparent power S_1:

$$\lambda_1 = \frac{|P_1|}{S_1}$$

According to 5.4.2, the fundamental power factor is also given by:

$$\lambda_1 = |\cos\varphi_1|$$

The fundamental power factor only refers to fundamental quantities.

NOTE The fundamental power factor is sometimes called displacement factor or displacement power factor.

5.5.3 Non-fundamental power factor

The non-fundamental power factor λ_N is defined as the ratio of the power factor λ to the fundamental power factor λ_1:

$$\lambda_N = \frac{\lambda}{\lambda_1}$$

Therefore, according to 5.5.1 and 5.5.2:

$$\lambda_N = \frac{|1 + (P_0 / P_1) + (P_D / P_1)|}{\sqrt{1 + DCR_U^2 + TDR_U^2} \times \sqrt{1 + DCR_I^2 + TDR_I^2}}$$

The non-fundamental power factor only refers to non-fundamental quantities: direct, harmonic and interharmonic components.

5.6 Summary

The definitions related to power quantities or indicators given in 5.3 to 5.5 are summarized in Table 1.

Table 1 – Summary of the power quantities under non-sinusoidal conditions

Quantity or indicator	Combined	Fundamental	Non-fundamental
Apparent power	S (VA)	S_1 (VA)	$\sqrt{S^2 - S_1^2}$ (VA)
Active power	P (W)	P_1 (W)	$P_0 + P_D$ (W)
Non active power	$\sqrt{S^2 - P^2}$ (var)	Q_1 (var)	$\sqrt{S^2 - S_1^2 - (P_0 + P_D)^2}$ (var)
Power factor	$\lambda = \|P\| / S$	$\lambda_1 = \|\cos\varphi_1\|$	$\lambda_N = \lambda / \lambda_1$

6 Electric power quantities with a sinusoidal voltage and a current distorted only with harmonics

6.1 Voltages and currents

6.1.1 Instantaneous values

In Clause 6, the voltage is assumed to be sinusoidal (this assumption can be made with a small error on power quantities when $TDR_{\mathrm{U}} < 5\ \%$; for more information, see 6.1.4, 6.3, 6.4.1 and 6.5.1) and the current assumed to have no d.c. component and to be distorted only with harmonic components. In this case, the reference fundamental frequency, which is the frequency of the a.c. power supply system, is equal to the fundamental frequency of the Fourier series. For steady-state conditions, the instantaneous values of the voltage and current are defined as follows:

$$u(t) = U_1\sqrt{2} \cdot \sin(\omega t + \alpha_1)$$

$$i(t) = I_1\sqrt{2} \cdot \sin(\omega t + \beta_1) + \sum_{h=2}^{\infty} I_h\sqrt{2} \cdot \sin(h\omega t + \beta_h)$$

where

$u(t)$ is the instantaneous value of the voltage at time t;

U_1 is the r.m.s. value of the voltage;

ω is the angular frequency corresponding to the fundamental frequency;

t is the time;

α_1 is the initial phase of the voltage;

$i(t)$ is the instantaneous value of the current at time t;

I_1 is the r.m.s. value of the fundamental component of the current;

β_1 is the initial phase of the fundamental component of the current;

h is the harmonic order (h is an integer number);

I_h is the r.m.s. value of the current harmonic component of order h;

β_h is the initial phase of the current harmonic component of order h.

The instantaneous value of the voltage or current can also be written as follows:

$$u(t) = u_1(t)$$

$$i(t) = i_1(t) + i_{\mathrm{H}}(t)$$

where

$u_1(t)$ is the fundamental component of the voltage at time t (see 6.1.2);

$i_1(t)$ is the fundamental component of the current at time t (see 6.1.2);

$i_{\mathrm{H}}(t)$ is the harmonic content of the current at time t (see 6.1.3).

6.1.2 Fundamental components

The fundamental component of the voltage (current) is defined as the sinusoidal component of the Fourier series of the voltage (current) having the frequency of the a.c. power supply system:

$$u_1(t) = U_1\sqrt{2} \cdot \sin(\omega t + \alpha_1)$$

$$i_1(t) = I_1\sqrt{2} \cdot \sin(\omega t + \beta_1)$$

6.1.3 Harmonic content of the current

The harmonic content of the current is defined as the sum of the harmonic components of the current:

$$i_H(t) = \sum_{h=2}^{\infty} I_h\sqrt{2} \cdot \sin(h\omega t + \beta_h)$$

6.1.4 RMS values of the voltage and current

The r.m.s. value of the voltage U (current I) is defined as the positive square root of the mean value of the square of the voltage (current) taken over an integer number of periods of the a.c. power supply system:

$$U = \sqrt{\frac{1}{kT}\int_{\tau}^{\tau+kT}[u(t)]^2 dt}$$

$$I = \sqrt{\frac{1}{kT}\int_{\tau}^{\tau+kT}[i(t)]^2 dt}$$

where

T is the reciprocal of the fundamental frequency;

k is an integer number;

τ is the time when the measurement starts.

These quantities can also be written as follows:

$$U = U_1$$

$$I^2 = I_1^2 + I_H^2$$

where

I_1 is the r.m.s. value of the fundamental component of the current;

I_H is the r.m.s. value of the harmonic content of the current (see 6.1.5).

NOTE The above formula, assuming that the voltage is sinusoidal, yields an error less than 0,15 % on the r.m.s. value of the voltage U, when $TDR_U < 5$ %.

6.1.5 RMS value of the harmonic content of the current

The r.m.s. value of the harmonic content of the current I_H can be written as follows:

$$I_H^2 = I^2 - I_1^2$$

According to 6.1.3, this quantity is also given by:

$$I_H^2 = \sum_{h=2}^{\infty} I_h^2$$

6.1.6 Total harmonic ratio of the current

The total harmonic ratio (also called total harmonic distortion) of the current THD_I is defined as the ratio of the r.m.s. value of its harmonic content to the r.m.s. value of its fundamental component:

$$THD_\mathrm{I} = \frac{I_\mathrm{H}}{I_1}$$

6.1.7 Fundamental factor

The fundamental factor of the current g_I is defined as the ratio of the r.m.s. value of its fundamental component to the r.m.s. value of the current itself:

$$g_\mathrm{I} = \frac{I_1}{I}$$

According to 6.1.4 and 6.1.6, the fundamental factor is also given by:

$$g_\mathrm{I} = \frac{1}{\sqrt{1 + THD_\mathrm{I}^2}}$$

6.2 Instantaneous power

The instantaneous power $p(t)$ is still defined as the product of the voltage $u(t)$ and the electric current $i(t)$:

$$p(t) = u(t) \cdot i(t)$$

6.3 Active power

The active power P is defined as the mean value, taken over an integer number of periods kT, of the instantaneous power $p(t)$:

$$P = \frac{1}{kT} \int_{\tau}^{\tau+kT} p(t)\, dt$$

In the case where the voltage waveform is sinusoidal, the active power reduces to its fundamental component:

$$P = U_1 I_1 \cos \varphi_1$$

where

φ_1 is the phase angle between the fundamental voltage and current components.

$$\varphi_1 = \alpha_1 - \beta_1$$

NOTE The above formula, assuming that the voltage is sinusoidal, yields an error less than a few per cent on the value of the active power P, when $TDR_\mathrm{U} < 5$ %. This error is highly dependent on the harmonic content of the current. It is generally less than 1 %, but can exceed 5 % in some exceptional cases.

6.4 Definitions related to the apparent power

6.4.1 Apparent power

The apparent power S is still defined as the product of the r.m.s. value of the voltage U and the r.m.s. value of the electric current I:

$$S = U \cdot I$$

NOTE 1 The apparent power corresponds to the maximum active power that can be transmitted to a load or system through a given line, while keeping the r.m.s. value of the voltage constant and the line losses constant. This maximum value of active power is obtained when the load or system is supplied with a sinusoidal current in phase with the voltage.

Using the separation of the r.m.s. current into its main components (see 6.1.4), the apparent power can also be written as follows:

$$S^2 = U_1^2 \cdot \left(I_1^2 + I_H^2\right)$$

$$S = S_1 \cdot \sqrt{1 + THD_I^2}$$

where

S is the apparent power;

S_1 is the fundamental apparent power (see 6.4.2);

THD_I is the total harmonic ratio of the current.

NOTE 2 The above formula, assuming that the voltage is sinusoidal, yields an error less than 0,15 % on the value of the apparent power S, when $TDR_U < 5$ %.

6.4.2 Fundamental apparent power

The fundamental apparent power S_1 is defined as the product of the r.m.s. values of the fundamental component of the voltage U_1 and the fundamental component of the current I_1:

$$S_1 = U_1 I_1$$

As for the particular case where the voltage and current are both sinusoidal, the fundamental apparent power has two components.

$$S_1^2 = P_1^2 + Q_1^2$$

where

P_1 is the fundamental active power (which is equal to the active power in this case);

Q_1 is the fundamental reactive power.

$$P_1 = U_1 I_1 \cos \varphi_1$$

$$Q_1 = U_1 I_1 \sin \varphi_1$$

6.5 Definitions related to the power factor

6.5.1 Power factor

The power factor λ is still defined as the ratio of the absolute value of the active power P to the apparent power S:

$$\lambda = \frac{|P|}{S}$$

NOTE 1 Since the apparent power is the maximum active power that can be transmitted to a load or system through a given line (see 6.4.1), the power factor is a utilization factor indicator.

According to 6.3, 6.4 and 6.1.7, the power factor can be written as follows:

$$\lambda = \frac{|U_1 I_1 \cos\varphi_1|}{U_1 I_1 \cdot \sqrt{1 + THD_I^2}} = g_I \cdot |\cos\varphi_1|$$

In this case, the power factor can still be written as the product of two factors:

$$\lambda = \lambda_1 \cdot \lambda_N$$

where

λ_1 is the fundamental power factor (see 6.5.2);

λ_N is the non-fundamental power factor (see 6.5.3).

NOTE 2 The above formulae, assuming that the voltage is sinusoidal, yield errors less than a few per cent on the values of the power factor λ and the non-fundamental power factor λ_N (see 6.5.3), when TDR_U < 5 %. This error is highly dependent on the harmonic content of the current. It is generally less than 1 %, but can exceed 5 % in some exceptional cases.

6.5.2 Fundamental power factor

The fundamental power factor λ_1 is still defined as the ratio of the absolute value of the fundamental active power P_1 to the fundamental apparent power S_1:

$$\lambda_1 = \frac{|P_1|}{S_1}$$

The fundamental power factor is also given by:

$$\lambda_1 = |\cos\varphi_1|$$

NOTE The fundamental power factor is sometimes called displacement factor or displacement power factor.

6.5.3 Non-fundamental power factor

The non-fundamental power factor λ_N is still defined as the ratio of the power factor λ to the fundamental power factor λ_1:

$$\lambda_N = \frac{\lambda}{\lambda_1}$$

Therefore:

$$\lambda_N = g_I = \frac{1}{\sqrt{1 + THD_I^2}}$$

In this case, the non-fundamental power factor is equal to the fundamental factor of the current.

NOTE See also Note 2 in 6.5.1.

6.6 Summary

The definitions related to power quantities or indicators given in 6.3 to 6.5 are summarized in Table 2.

Table 2 – Summary of the power quantities with a sinusoidal voltage and a current distorted only with harmonics

Quantity or indicator	Combined	Fundamental	Non-fundamental
Apparent power	S (VA)	S_1 (VA)	$\sqrt{S^2 - S_1^2}$ (VA)
Active power	P (W)	P_1 (W)	0
Non active power	$\sqrt{S^2 - P^2}$ (var)	Q_1 (var)	$\sqrt{S^2 - S_1^2}$ (var)
Power factor	$\lambda = \lvert P \rvert / S$	$\lambda_1 = \lvert \cos \varphi_1 \rvert$	$\lambda_N = g_I = \frac{1}{\sqrt{1 + THD_I^2}}$

Annex A
(normative)

Electric power quantities under sinusoidal conditions

A.1 Instantaneous values of the voltage and current

In Annex A, the case of a linear load supplied by a sinusoidal voltage source is considered. The current flowing through the load is thus also sinusoidal. The instantaneous values of the voltage and current can be defined as follows:

$$u(t) = U\sqrt{2} \cdot \sin(\omega t + \varphi)$$

$$i(t) = I\sqrt{2} \cdot \sin(\omega t)$$

where

$u(t)$ is the instantaneous value of the voltage at time t;

U is the r.m.s. value of the voltage;

ω is the angular frequency corresponding to the fundamental frequency;

t is the time;

φ is the initial phase of the voltage (in Annex A, φ is also the displacement angle; see 3.13);

$i(t)$ is the instantaneous value of the current at time t;

I is the r.m.s. value of the current;

The voltage leads the current when $\varphi > 0$ and lags the current when $\varphi < 0$ (see Figures A.1 and A.2).

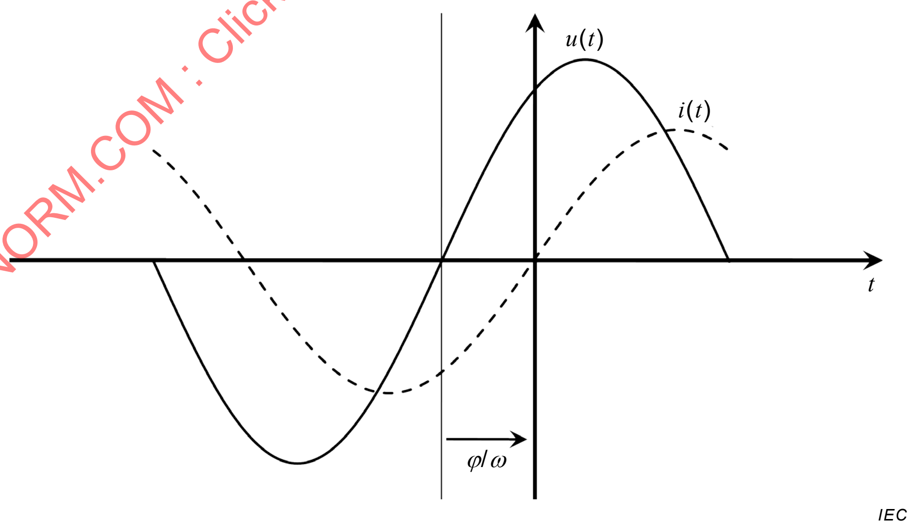

Figure A.1 – Illustration of the displacement angle (φ) when the voltage leads the current, $\varphi > 0$