INTERNATIONAL STANDARD

ISO 22009

First edition
2009-12-01

Space systems — Space environment (natural and artificial) — Model of the earth's magnetospheric magnetic field

Systèmes spatiaux — Environnement spatial (naturel et artificiel) — Modèle du champ magnétique de la magnétosphère de la terre

Reference number
ISO 22009:2009(E)

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Foreword

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ISO 22009 was prepared by Technical Committee ISO/TC 20, *Aircraft and space vehicles*, Subcommittee SC 14, *Space systems and operations*.

Introduction

This International Standard describes the main requirements to the earth's magnetospheric magnetic field model. The model satisfying the set of requirements is described in Annex A as example calculations. The model can be used in scientific and engineering applications and is intended to calculate the magnetic induction field generated from a variety of current systems located on the boundaries and within the boundaries of the earth's magnetosphere under a wide range of environmental conditions, quiet and disturbed, that are affected by solar-terrestrial interactions stimulated by solar activity such as solar flares and related phenomena, which induce terrestrial magnetic disturbances such as magnetic storms.

Space systems — Space environment (natural and artificial) — Model of the earth's magnetospheric magnetic field

1 Scope

This International Standard describes the main magnetospheric large-scale current systems and the magnetic field in the earth's magnetosphere and provides the main requirements for a model of the magnetospheric magnetic field. Ionospheric currents are not considered in this International Standard. Annex A of this International Standard gives a worked example of the model and establishes the parameters of magnetospheric large-scale current systems that change according to conditions in the space environment. This International Standard can be used to develop new models of the magnetospheric magnetic field. Such models are useful in investigating the physical processes in the earth's magnetosphere as well as in calculations associated with developing, testing and estimating the results of exploitation of spacecrafts and other equipment operating in the space environment.

The main goals of standardizing the concepts of the earth's magnetospheric magnetic field are to provide

— an unambiguous presentation of the magnetic field in the earth's magnetosphere;

— compatibility for the results of the interpretation and analysis of space experiments;

— less labour-intensive calculations of the magnetic field of magnetospheric currents in space at geocentric distances of 1,0 to 6,6 earth radii, R_E;

— the most reliable calculations of all elements of the geomagnetic field in the space environment.

The magnetic field model presented in Annex A of this International Standard can be used to predict the radiation conditions in space, including the periods of intense magnetic disturbances (magnetic storms), when developing systems of spacecraft magnetic orientation and when forecasting the influence of magnetic disturbances on transcontinental piping and power transmission lines.

2 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

2.1
internal magnetic field
⟨main magnetic field⟩ magnetic field produced by the sources inside the earth's core

NOTE It can be presented in the form of a series of spherical harmonic functions. The expansion coefficients [International Geomagnetic Reference Field (IGRF) model] undergo very slight changes in time. The International Association of Geomagnetism and Aeronomy (IAGA) is responsible for IGRF model development and modifications and approves its coefficients every five years. Internal magnetic field is not addressed by this International Standard.

2.2
external magnetic field
⟨magnetospheric magnetic field⟩ magnetic field produced by magnetospheric sources of magnetic field

2.3
magnetospheric sources of magnetic field
sources of magnetic fields including the following:

— currents flowing over the magnetopause and screening the geomagnetic dipole magnetic field;

— currents flowing inside the earth's magnetosphere, including

  — tail currents, produced by currents across the geomagnetic tail and closure currents on the magnetopause,

  — ring currents, including symmetrical ring current, circling around the earth and carried by trapped particles and partial ring current, produced by azimuthal currents at low latitudes flowing mostly in the pre-midnight sector, and closed by field-aligned and ionospheric currents,

  — field-aligned currents, produced by currents flowing along the high-latitude magnetic field lines, closed by currents on the magnetopause and in the ionosphere;

— currents flowing over the magnetopause and screening the ring current and partial ring current magnetic fields

NOTE 1 Electric currents flowing entirely in the ionosphere (ionospheric currents) contribute to the magnetic field variation at altitudes below 1 000 km. In the region above 1,5 R_E, the effect of the ionospheric current is insignificant.

NOTE 2 Magnetic field of ionospheric currents is not addressed by this International Standard.

2.4
geomagnetic dipole tilt angle
angle of inclination of the geomagnetic dipole to the plane orthogonal to the earth-sun line

2.5
solar-magnetospheric (GSM) coordinates
Cartesian geocentric coordinates, where the X-axis is directed to the sun, the Z-axis, which is orthogonal to the X-axis, lies on the plane with the X-axis and the geomagnetic dipole axis, and the Y-axis supplements the X-and Z-axes to the right-hand system

2.6
magnetopause stand-off distance
geocentric distance to the subsolar point on the magnetopause

3 General concepts and assumptions

3.1 Magnetic field induction in the earth's magnetosphere

The vector, $\vec{B}_M$, expressed in nanoteslas, of the magnetic field induction in the earth's magnetosphere is calculated from Equation (1):

$$\vec{B}_M = \vec{B}_1 + \vec{B}_2 \tag{1}$$

where

$\vec{B}_1$ is the vector of induction of the internal magnetic field;

$\vec{B}_2$ is the vector of induction of the external magnetospheric magnetic field.

The magnetic field of the magnetospheric currents (external magnetic field), $\vec{B}_2$, is calculated in terms of the quantitative model of the magnetosphere.

3.2 Magnetospheric magnetic field standardization: process-based approach

This International Standard for magnetospheric magnetic fields does not specify a single magnetospheric model, theoretical or empirical. In order to encourage continual improvements in magnetospheric modelling, this International Standard is a process-based standard for determining the magnetospheric magnetic field. The magnetospheric magnetic field model, after its development, can satisfy the requirements in Clause 4 and the list of criteria presented in Clause 5. The worked example of the model is presented in Annex A and it is necessary that it be reconsidered every five years relative to the candidate models. The current worked example is presented in the Annex A.

4 Model requirements

4.1 General

The model of the magnetic field of magnetospheric currents (subsequently referred to as the “model”) presents the vector of induction of magnetospheric currents in solar-magnetospheric coordinates.

The model describes a regular part of the magnetic field in the region from 1,0 R_E to 6,6 R_E.

The model reflects the compression of the earth's magnetosphere in the dayside due to interaction with the solar wind, day-night asymmetry (i.e. the field on the nightside is weakened), daily and seasonal variations.

The model takes into account the geomagnetic dipole tilt angle, varying in the range from –35° to +35°.

4.2 Magnetospheric magnetic field sources

The standardized magnetospheric magnetic field is produced by the currents described in 2.3. The effect of the ionospheric currents is not addressed by this International Standard.

4.3 Parameterization

Each magnetospheric source of magnetic field depends on parameters that are calculated from empirical data.

4.4 Magnetospheric dynamics

The magnetospheric dynamics is determined from a sequence of its instantaneous states.

4.5 Model testing and comparison with measurements

The model testing is carried out with the help of databases that include spacecraft-based and on-ground measurements. The dataset used for the model testing is presented in Annex C.

5 List of criteria

The compliance criteria for this International Standard consist of the activities common for any candidate magnetospheric magnetic field model. These criteria specify the compliance process that includes the documentation, publication and testing of the model and include the following.

— The candidate model shall include a statement of the modeling approach used (empirical or theoretical model). The empirical models shall include a clear specification of the input data used to derive the model and where these data were measured. Theoretical models shall include a description of the physical principles and approaches that are used as the basis of the model.

- A statement concerning the candidate model area of application and domain of applicability should be included.
- A statement about the root-mean-square (rms) errors during the model calculations relative to the deviations from the observational data obtained from measurements should be included. For empirical models, comparisons should also be made with data, different from those on which the model is based.
- A description and implementation of the magnetospheric magnetic field model should be published in internationally accessible, refereed journals.

Annex A
(informative)

Paraboloid model of the magnetospheric magnetic field: calculation of induction of the magnetic field of the magnetospheric currents

A.1 Paraboloid model of the magnetic field of magnetospheric currents

A.1.1 General

The magnetospheric magnetic field calculated by paraboloid model is a solution of the magnetostatic problem inside the paraboloid of revolution.

The vector of induction of the magnetic field of magnetospheric currents is calculated from Equation (A.1):

$$\vec{B}_2 = \vec{B}_{sd}\left(\psi, R_1\right) + \vec{B}_t\left(\psi, R_1, R_2, \Phi_\infty\right) + \vec{B}_r\left(\psi, b_t\right) + \vec{B}_{sr}\left(\psi, R_1, b_r\right) + \vec{B}_{fac}\left(I_0\right) \quad \text{(A.1)}$$

where

$\vec{B}_{sd}$ is the magnetic field of currents on the magnetopause screening the dipole field;

$\vec{B}_t$ is the magnetic field of the magnetospheric tail;

$\vec{B}_r$ is the magnetic field of the ring current;

$\vec{B}_{sr}$ is the magnetic field of currents on the magnetopause, screening the ring current field;

$\vec{B}_{fac}$ is the magnetic field of region 1 field-aligned currents.

The components of the magnetic field of magnetospheric currents, $\vec{B}_{sd}$, $\vec{B}_t$, $\vec{B}_r$, $\vec{B}_{sr}$ and $\vec{B}_{fac}$ are calculated separately in terms of the paraboloid model of the magnetosphere in the form of series in the Bessel functions or Legendre polynomials.

A.1.2 Parameters

The components of the magnetic field of magnetospheric currents, $\vec{B}_{sd}$, $\vec{B}_t$, $\vec{B}_r$, $\vec{B}_{sr}$ and $\vec{B}_{fac}$ are determined from the values of the following parameters of the magnetospheric current systems:

ψ is the geomagnetic dipole tilt angle, expressed in degrees;

$\vec{R}_1$ is the distance to the subsolar point at the magnetopause, expressed in earth radii;

$\vec{R}_2$ is the distance to the earthward edge of the magnetospheric tail current sheet, expressed in earth radii;

Φ_∞ is the magnetic flux in the tail lobes, defining the current intensity in the magnetotail, expressed in webers;

b_r is the intensity of the ring current magnetic field at the earth's centre, expressed in nanoteslas;

I_0 is the total region 1 field-aligned currents intensity, expressed in mega-amperes.

A.1.3 Submodels

The instantaneous values of the parameters of the magnetospheric current systems, ψ, $\vec{R}_1$, $\vec{R}_2$, Φ_∞, b_r and I_0 are determined using a limited set of empirical data in terms of the so-called submodels (see Annex B).

A.2 Magnetic field of the magnetopause currents screening the geomagnetic dipole

$\vec{B}_{sd}$ is calculated as given by Equation (A.2):

$$\vec{B}_{sd} = \nabla U_{sd} \tag{A.2}$$

where the scalar potential, U_{sd}, of the magnetic field of the magnetopause currents is presented in the spherical coordinates R, θ and φ (see C.1), as given in Equation (A.3):

$$U_{sd} = -\frac{M_E}{R_1^2} \sum_{n=1}^{\infty} \left(\frac{R}{R_1} \right)^n \left[d_n^{||} \sin\psi \cdot P_n(\cos\theta) + d_n^{\perp} \cos\psi \cos\varphi \cdot P_n^1(\cos\theta) \right] \tag{A.3}$$

where P_n is given by Equation (A.4):

$$P_n = \left(2^n n!\right)^{-1} \cdot \left[\mathrm{d}^n \left(x^2 - 1\right)^n \Big/ \mathrm{d}x^n \right], \; P_n^1(x) = \sqrt{1 - x^2} \cdot \left(\mathrm{d}P_n / \mathrm{d}x\right) \tag{A.4}$$

The magnetic moment of the geomagnetic dipole, M_E, is given by Equation (A.5):

$$M_E = B_0 \cdot R_E^3 \tag{A.5}$$

where B_0 is the magnetic field at the geomagnetic equator of the earth.

The first six dimensionless coefficients $d_n^{||}$ and $d_n^{\perp}$ are listed in the Table A.1.

Table A.1 — Expansion coefficients for the scalar potential of the magnetic field of magnetopause currents

n	$d_n^{\perp}$	$d_n^{||}$
1	0,649 7	0,940 3
2	0,216 5	0,465 0
3	0,043 4	0,129 3
4	−0,000 8	−0,014 8
5	−0,004 9	−0,016 0
6	−0,002 2	−0,022 5

A.3 Magnetic field of the tail current system

The magnetic field of the tail current system, $\vec{B}_{\mathrm{t}}$, is calculated from Equation (A.6):

$$\vec{B}_{\mathrm{t}} = -\nabla U_{\mathrm{t}} + \vec{B}_{\mathrm{t,in}} \tag{A.6}$$

where U_{t} is determined by Equations (A.7) to (A.10):

$$U_{\mathrm{t}} = b_{\mathrm{t}}\, R_1 \begin{cases} \sum\limits_{k,n=1}^{\infty} \left[b_{nk} + c_{nk} K_n'(\lambda_{nk}\alpha_0)\lambda_{nk} \right] \cos n\varphi \cdot J_n(\lambda_{nk}\beta) I_n(\lambda_{nk}\alpha) & \text{for } \alpha < \alpha_0 \\ \beta_t \alpha_0 \ln\alpha \cdot \mathrm{sign}\left(\frac{\pi}{2} - |\varphi|\right) + \ldots & \\ \ldots \sum c_{nk} \cos n\varphi \cdot I_n'(\lambda_{nk}\alpha_0)\lambda_{nk} J_n(\lambda_{nk}\beta) K_n(\lambda_{nk}\alpha) & \text{for } \alpha \geqslant \alpha_0 \end{cases} \tag{A.7}$$

where

$$c_{nk} = b_{nk}\, \lambda_{nk}\, I_n\left(\lambda_{nk}\alpha_0\right) \tag{A.8}$$

$$b_{nk} = \frac{2\lambda_{nk} \int\limits_0^1 \int\limits_{-\pi}^{\pi} J_n\left(\lambda_{nk}\beta\right) f\left(\beta, \varphi\right) \cos n\varphi \, d\varphi \, d\beta}{\pi\left(\lambda_{nk}^2 - n^2\right) J_n^2\left(\lambda_{nk}\right) I_n'\left(\lambda_{nk}\alpha_0\right)} \tag{A.9}$$

$$f\left(\beta, \varphi\right) = \begin{cases} \frac{\alpha_0}{\beta_{\mathrm{t}}}\beta\cos\varphi, & \text{for } \alpha_0\, \beta \cos\varphi < \beta_{\mathrm{t}} \\ \mathrm{sign}\left(\frac{\pi}{2} - |\varphi|\right), & \text{for } \alpha_0\, \beta \cos\varphi \geqslant \beta_{\mathrm{t}} \end{cases} \tag{A.10}$$

where

λ_{nk} is the zero of the equation $J' = 0$;

α_0 is the parabolic α coordinate of the inner edge of the tail current sheet, equal to $\sqrt{1 - 2R_2/R_1}$;

$\beta_{\mathrm{t}} = \frac{d}{R_1}$

d is the half thickness of the current sheet;

b_{t} is the magnetic field in the tail lobe at the inner edge of the tail current sheet, equal to $\frac{2\Phi_\infty}{\pi R_1^2}\sqrt{R_1/(2R_2 + R_1)}$.

The magnetic field inside the current sheet, $\vec{B}_{\mathrm{t,in}}$, is calculated from Equation (A.11):

$$\vec{B}_{\mathrm{t,in},\alpha} = b_{\mathrm{t}} \frac{\alpha_0}{\alpha} \frac{\beta}{\beta_{\mathrm{t}}} \frac{\cos\varphi}{\sqrt{\alpha^2 + \beta^2}},\ \vec{B}_{\mathrm{t,in},\beta} = 0,\ \vec{B}_{\mathrm{t,in},\varphi} = 0 \tag{A.11}$$

A description of the paraboloid coordinates is presented in C.3.

A.4 Ring current magnetic field

The ring current magnetic field, $\vec{B}_r$, is determined by Equation (A.12):

$$\vec{B}_r = \frac{M_R}{M_E} \cdot \begin{cases} \left[\left(\frac{R}{R_{rc}}\right)^5 \cdot \vec{B}_d + 2B_0 \frac{R_E^3}{R_2^3}\left(\frac{R_2^5}{R_{rc}^5} - 1\right)\vec{e}_z\right] & \text{for } 0 \leqslant R \leqslant R_2 \\ \vec{B}_d & \text{for } R \geqslant R_2 \end{cases} \quad \text{(A.12)}$$

where

$R_{rc} = \sqrt{0{,}5\left(R^2 + R_2^2\right)}$;

M_R is the magnetic moment of the ring current, equal to $0{,}5\, b_r \cdot R_2^3 / \left(4\sqrt{2} - 1\right)$;

$\vec{B}_d$ is the magnetic field of the geomagnetic dipole;

$\vec{e}_z$ is a unit vector in the direction opposite to the geomagnetic dipole.

Expressions for $\vec{B}_d$ and $\vec{e}_z$ in the solar-magnetospheric coordinates are presented in C.4.

A.5 Magnetic field of the magnetopause currents screening the ring current

The magnetic field of the magnetopause currents screening the ring current, $\vec{B}_{sr}$, is calculated from Equation (A.13):

$$\vec{B}_{sr} = -\nabla U_{sr} \quad \text{(A.13)}$$

where the scalar potential, U_{sr}, of the magnetic field of magnetospheric currents presented in spherical coordinates R, θ and φ (see C.2) is given by Equation (A.14):

$$U_{sr} = -\frac{M_R}{M_1^2} \sum_{n=1}^{\infty} \left(\frac{R}{R_1}\right)^n \left[d_n^{||} \sin\psi \cdot P_n(\cos\theta) + d_n^{\perp} \cos\psi \cos\varphi \cdot P_n^1(\cos\theta)\right] \quad \text{(A.14)}$$

The coefficients $d_n^{||}$ and $d_n^{\perp}$ are listed in Table A.1.

A.6 Magnetic field of field-aligned currents

The magnetic field of field-aligned currents, $\vec{B}_{fac}$, is calculated from Equation (A.15):

$$\vec{B}_{fac} = \text{curl}\, \vec{A}_{fac} \quad \text{(A.15)}$$

where the vector potential $\vec{A}_{fac}$ of the magnetic field of field-aligned currents is presented in spherical coordinates R, θ and φ with the polar axis in a direction opposite to the earth's dipole (see C.2 and C.4), as given in Equation (A.16):

$$\vec{A}_{\text{fac}} = \frac{\mu_0 I_0 \sin\varphi}{2(1+\cos\theta_{\text{m}})} \begin{cases} \dfrac{\tan(\theta/2)}{\tan(\theta_{\text{m}}/2)} & \text{for} \quad 0 \leqslant \theta \leqslant \theta_{\text{m}} \\ \dfrac{\sin\theta_{\text{m}}}{\sin\theta} & \text{for} \quad \theta_{\text{m}} \leqslant \theta \leqslant \pi - \theta_{\text{m}} \\ \dfrac{\cot(\theta/2)}{\tan(\theta_{\text{m}}/2)} & \text{for} \quad \pi - \theta_{\text{m}} \leqslant \theta \leqslant \pi \end{cases} \tag{A.16}$$

where θ_{m} is the polar cap radius, expressed in radians, as given by Equation (A.17):

$$\sin^2 \theta_{\text{m}} = 3{,}9 \cdot \left(\Phi_{\infty} / |B_0| \right) \tag{A.17}$$

where Φ_{∞} is expressed in megawebers and B_0 in nanoteslas.

A.7 Accuracy of the model

A.7.1 Comparison with the large magnetosphere magnetic field database

NOTE See Reference [1].

Analysis of the distribution of the relative discrepancies integral over the whole experimental material indicates a discrepancy mean value of about + 3 %; σ of the distribution is about 80 %.

A.7.2 Comparisons with Dst and satellite measurements

RMS errors are about 10 % to 15 % of the peak worldwide magnetic storm level (Dst) for different magnetic storms.

A comparison between the model calculations and the empirical data is presented in detail in the Explanatory Report and published in References [2] to [5].

A.8 Other relevant models

Other relevant models include the following:

a) semi-empirical T96 model; see Reference [6];

b) semi-empirical T01 model; see References [7] and [8];

c) semi-empirical T04 model; see Reference [9].

Parameterization for the models is performed using large magnetospheric databases and is different for different models. Several model revisions reflect the different mathematical description of the major sources of the magnetospheric field and their different parameterization. The most popular T01 model parameters are the geomagnetic dipole tilt angle, interplanetary magnetic field (IMF) B_y and B_z components, solar wind dynamic pressure and Dst index. An attempt is made to take into account the prehistory of the solar wind by introducing two functions, G1 and G2, that depend on the IMF B_z and solar wind velocity and their time history.

Annex B
(informative)

Submodels

B.1 Submodels — Calculation of the main parameters of magnetospheric current systems

In the paraboloid model of the magnetosphere, the values of the parameters of the magnetospheric current systems are calculated using submodels. The submodels represent empirical correlations or auxiliary models to relate the parameters of the magnetospheric current systems to the measured data. While the magnetic field dependence on parameters is fixed, the parameters' dependence on empirical data can be changed by the model's user. Below are the simple submodels that allow a calculation of the model input parameters. These submodels are not subject to standardization.

B.2 Tilt angle of the geomagnetic dipole

The tilt angle of the geomagnetic dipole, ψ, is calculated from Equation (B.1):

$$\sin\psi = -\sin\beta\cos\alpha_1 + \cos\beta\sin\alpha_1\cos\varphi_m \quad \text{(B.1)}$$

where

α_1 is the angle between the earth's axis and the geomagnetic dipole moment and is equal to 11,43°;

β is the sun's deflection, defined as $\sin\beta = \sin\alpha_2\cos\varphi_{se}$;

α_2 is the angle between the earth's axis and the normal to the ecliptic plane and is equal to 23,5°;

φ_{se} is the angle between the earth-sun line and the projection of the earth's axis at the ecliptic plane and is equal to $0{,}985\,626\,3\,(172 - I_{day})$;

I_{day} is the number of the day in a year;

φ_m is the angle between the midnight geographic meridian plane and northern magnetic pole meridian plane and is equal to $T_U \cdot 15° - 69{,}76°$;

T_U is the universal time, expressed in hours.

B.3 Distance from the earth to the subsolar point on the magnetopause

The geocentric distance, R_1, to the subsolar point is calculated using solar wind data: the solar wind dynamical pressure and IMF B_z component as given by Equation (B.2) (see Reference [10]):

$$R_1 = \left\{10{,}22 + 1{,}29\tanh\left[0{,}184\left(B_z + 8{,}14\right)\right]\right\}\left(nv^2\right)^{-\frac{1}{6{,}6}} \quad \text{(B.2)}$$

where

B_z is the IMF z component, expressed in nanoteslas;

n is the solar wind concentration, expressed in reciprocal cubic centimetres;

v is the solar wind velocity, expressed in kilometres per second.

B.4 Distance to the earthward edge of the geomagnetic tail current sheet

The distance to the earthward edge of the geomagnetic tail current sheet, R_2, is calculated as given in Equation (B.3)

$$R_2 = 1/\cos^2 \varphi_k \tag{B.3}$$

where

R_2 is expressed in earth radii;

φ_k is the latitude of the equatorward boundary of the auroral oval at midnight.

B.5 Magnetic flux through the magnetotail lobes

Magnetic flux through the magnetotail lobes, Φ_∞, is calculated as given in Equation (B.4):

$$\Phi_\infty = \Phi_0 + \Phi_s \tag{B.4}$$

where

Φ_0 is the magnetic flux in the magnetotail during quiet periods, equal to $3{,}7 \cdot 10^8$ Wb;

Φ_s the time-dependent magnetic flux in the lobes associated with intensification of the magnetotail current system during disturbances, equal to $-A_L \dfrac{\pi R_1^2}{14} \sqrt{\dfrac{2R_2}{R_1} + 1}$

where A_L is the auroral index of geomagnetic activity; see References [2] and [4].

B.6 Ring current magnetic field at the earth's centre

The ring current intensity is characterized by the value of ring current magnetic field at the earth's centre, which is calculated by the Dessler-Parker-Scopke relation, as given in Equation (B.5):

$$b_r = -\frac{2}{3} B_0 \frac{\varepsilon_r}{\varepsilon_d} \tag{B.5}$$

where

ε_r is the total energy of ring current particles;

ε_d is the geomagnetic dipole energy, equal to $\frac{1}{3} B_0 M_E$; see References [2] and [11].