TECHNICAL REPORT

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Statistical methods in process management — Capability and performance —

Part 4: Process capability estimates and performance measures

Méthodes statistiques dans la gestion de processus — Aptitude et performance —

Partie 4: Estimations de l'aptitude de processus et mesures de performance

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 22514-4 has been prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

ISO 22514 consists of the following parts, under the general title *Statistical methods in process management — Capability and performance*:

— *Part 1: General principles and concepts*

— *Part 3: Machine performance studies for measured data on discrete parts*

— *Part 4: Process capability estimates and performance measures* [Technical Report]

In the future, it is planned to revise ISO 21747:2006 (*Statistical methods — Process performance and capability statistics for measured quality characteristics*) as Part 2.

NOTE ISO/TR 22514-4 was initially prepared as ISO/DTR 12783. It was renumbered before publication to include it in the ISO 22514 series.

Introduction

Many organizations have embarked upon a continuous improvement strategy. To comply with such a strategy, any organization will need to evaluate the capability and performance of its key processes. The methods described in this part of ISO 22514 are intended to assist any management in this respect. These evaluations need to be constantly reviewed by management so that actions compatible with continuous improvement can be taken when required.

The content of this part of ISO 22514 has been subject to large shifts of opinion during recent times and this report attempts to reflect the current position. The most fundamental shift has been to philosophically separate what is named in this document as capability conditions from performance conditions, the primary difference being whether statistical stability has been obtained (capability) or not (performance). This naturally leads onto the two sets of indices that are to be found in their relevant clauses. It has become necessary to draw a firm distinction between these since it has been observed in industry that companies have been deceived about their true capability position due to inappropriate indices being calculated and published.

The progression of this part of ISO 22514 is from the general condition to the specific and this approach leads to general formulae being presented before their more usual but specific manifestations.

There exist numerous references that describe the importance of understanding the processes at work within any organization, be it a manufacturing process or an information handling process. As organizations compete for sales with each other, it has become increasingly apparent that it is not only the price paid for a product or service that matters so much, but also what costs will be incurred by the purchaser from using such a product or service. The objective for any supplier is to continually reduce variability and not to just satisfy specification.

Continual improvement leads to reductions in the costs of failure and assists in the drive for survival in an increasingly more competitive world. There will also be savings in appraisal costs for as variation is reduced the need to inspect product might disappear or the frequency of sampling might be reduced.

Process capability and performance evaluations are necessary to enable organizations to assess the capability and performance of their suppliers. Those organizations will find the indices contained within this part of ISO 22514 useful in this endeavour.

Quantifying the variation present within a process enables judgement of its suitability and ability to meet some given requirement. The following paragraphs and clauses provide an outline of the philosophy required to be understood to determine the capability or performance of any process.

All processes will be subject to certain inherent variability. This part of ISO 22514 does not attempt to explain what is meant by inherent variation, why it exists, where it comes from nor how it affects a process. This part of ISO 22514 starts from the premise that it exists and is stable.

Process owners should endeavour to understand the sources of variation in their processes. Methods such as flowcharting the process and identifying the inputs and outputs from a process assist in identification of these variations together with the appropriate use of cause and effect (fishbone) diagrams.

It is important for the user of this part of ISO 22514 to appreciate that variations exist that will be of a short-term nature as well as those that will be of a long-term nature and that capability determinations using only the short-term variation might be greatly different to those which have used the long-term variability.

When considering short-term variation, a study that uses only the shortest-term variation, sometimes known as a machine study, might be carried out. The method required to carry out such a study will be outside the scope of this part of ISO 22514; however, it should be noted that such studies are important and useful.

It should be noted that where the capability indices given in this part of ISO 22514 are computed, they only form point estimates of their true values. It is therefore recommended that, wherever possible, the indices' confidence intervals are computed and reported. This part of ISO 22514 describes methods by which these can be computed.

Statistical methods in process management — Capability and performance —

Part 4:
Process capability estimates and performance measures

1 Scope

This part of ISO 22514 describes process capability and performance measures that are commonly used.

2 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

2.1 Basic terms

2.1.1
product
result of a process

NOTE Process is defined in ISO 12207:1995, 3.17 and in ISO 9000:2005, 3.4.1.

2.1.2
characteristic
distinguishing feature

NOTE 1 A characteristic can be inherent or assigned.

NOTE 2 A characteristic can be qualitative or quantitative.

NOTE 3 There are various classes of characteristics, such as the following:

— physical (e.g. mechanical, electrical, chemical or biological characteristics);

— sensory (e.g. relating to smell, touch, taste, sight, hearing);

— behavioural (e.g. courtesy, honesty, veracity);

— temporal (e.g. punctuality, reliability, availability);

— ergonomic (e.g. physiological characteristic, or related to human safety);

— functional (e.g. maximum speed of an aircraft).

[ISO 9000:2005, 3.5.1]

2.1.3
quality characteristic
inherent **characteristic** (2.1.2) of a **product** (2.1.1), process or system related to a requirement

NOTE 1 Inherent means existing in something, especially as a permanent characteristic.

NOTE 2 A characteristic assigned to a product, process or system (e.g. the price of a product, the owner of a product) is not a quality characteristic of that product, process or system.

[ISO 9000:2005, 3.5.2]

2.1.4
specification limit
limiting value stated for a **characteristic** (2.1.2)

[ISO 3534-2:2006, 3.1.3]

2.1.5
specified tolerance
difference between the upper specification limits and lower **specification limits** (2.1.4)

[ISO 3534-2:2006, 3.1.6]

2.1.6
target value
T
preferred or reference value of a **characteristic** (2.1.2) stated in a specification

[ISO 3534-2:2006, 3.1.2]

2.1.7
distribution
⟨of a characteristic⟩ information on the probabilistic behaviour of a **characteristic** (2.1.2)

NOTE 1 The distribution of a **characteristic** (2.1.2) can be represented, for example, by ranking of the values of the **characteristic** (2.1.2) and showing the resulting pattern of measures or scores in the form of a tally chart or histogram. Such a pattern provides all of the numerical value information of the **characteristic** (2.1.2) except for the serial order in which the data arises.

NOTE 2 The distribution of a **characteristic** (2.1.2) is dependent on prevailing conditions. Thus, if meaningful information about the distribution of a **characteristic** (2.1.2) is desired the conditions under which the data are collected should be specified.

NOTE 3 It is important to know the **class of distributions** (2.1.8), for instance, normal or log-normal, before predicting or estimating process capability or performance measures and indices or fraction nonconforming.

[ISO 3534-2:2006, 2.5.1]

2.1.8
class of distributions
particular family of **distributions** (2.1.7) each member of which has the same common attributes by which the family is fully specified

EXAMPLE 1 The two-parameter, symmetrical bell-shaped, normal distribution with parameters mean and standard deviation.

EXAMPLE 2 The three-parameter Weibull distribution with parameters location, shape and scale.

EXAMPLE 3 The unimodal continuous distributions.

NOTE 1 The class of distributions can often be fully specified through the values of the appropriate parameters.

NOTE 2 Tests for the normality of data can be found in ISO 5479.

NOTE 3 Adapted from ISO 3534-2:2006, 2.5.2.

2.1.9
distribution model
specified **distribution** (2.1.7) or **class of distributions** (2.1.8)

EXAMPLE 1 A model for the distribution of a product characteristic, such as the diameter of a bolt, might be the normal distribution with mean 15 mm and standard deviation 0,05 mm. Here the model is a fully specified one.

EXAMPLE 2 A model for the diameter of bolts as in Example 1 could be the class of normal distributions without attempting to specify a particular distribution. Here the model is the class of normal distributions.

[ISO 3534-2:2006, 2.5.3]

2.1.10
reference limits
nominated quantiles of the **distribution** (2.1.7) of the product characteristic

NOTE 1 The conditions of the **distribution** (2.1.7) of the product characteristic must be specified, see Notes 2 and 3 of 2.1.7.

NOTE 2 Traditionally the 0,135 % and 99,865 % quantiles have been used.

EXAMPLE If the **distribution** (2.1.7) of the product characteristic is normal with mean μ and standard deviation σ, the limits are $\mu \pm 3\sigma$ when the traditional 0,135 % and 99,865 % quantiles are used.

2.1.11
reference interval
interval bounded by the 99,865 % distribution quantile, $X_{99,865\,\%}$, and the 0,135 % distribution quantile, $X_{0,135\,\%}$

NOTE 1 The interval can be expressed by $\left(X_{99,865\,\%}, X_{0,135\,\%}\right)$ and the length of the interval is $\left(X_{99,865\,\%} - X_{0,135\,\%}\right)$.

NOTE 2 This term is used only as an arbitrary, but standardized, basis for defining the **process performance index** (2.3.3) and **process capability index** (2.2.3).

NOTE 3 For a normal **distribution** (2.1.7), the reference interval may be expressed in terms of six standard deviations, 6σ, or $6S$, when estimated from a sample.

NOTE 4 For a non-normal distribution, the length of the reference interval can be estimated by means of appropriate probability papers (e.g. log-normal) or from the sample kurtosis and sample skewness using, for example, Pearson curves.

NOTE 5 A quantile or fractile indicates division of a distribution into equal units or fractions, e.g. percentiles. Quantile is defined in ISO 3534-1.

[ISO 3534-2:2006, 2.5.7]

2.1.12
upper fraction nonconforming
p_U
fraction of the **distribution** (2.1.7) of a **characteristic** (2.1.2) that is greater than the upper **specification limit** (2.1.4), U

EXAMPLE In a normal **distribution** (2.1.7), with mean, μ, and standard deviation σ:

$$p_U = 1 - \Phi\left(\frac{U-\mu}{\sigma}\right) = \Phi\left(\frac{\mu-U}{\sigma}\right)$$

where

p_U is the upper fraction nonconforming;

Φ is the distribution function of the standard normal distribution;

U is the upper specification limit.

NOTE 1 Tables (or functions in statistical computer packages) of the standard normal distribution are readily available which give the proportion of process output expected beyond a particular value of interest, such as a **specification limit** (2.1.4), in terms of standard deviations away from the process mean. This obviates the need to work out the statistical distribution function given in the example.

NOTE 2 The function relates to a theoretical distribution. In practice, with empirical distributions, the parameters are replaced by estimates.

[ISO 3534-2:2006, 2.5.4]

2.1.13
lower fraction nonconforming
p_L
fraction of the **distribution** (2.1.7) of a **characteristic** (2.1.2) that is less than the lower **specification limit** (2.1.4), L

EXAMPLE In a normal **distribution** (2.1.7), with mean, μ, and standard deviation, σ:

$$p_L = \Phi\left(\frac{L-\mu}{\sigma}\right)$$

where

p_L is the lower fraction nonconforming;

Φ is the distribution function of the standard normal distribution;

L is the lower specification limit.

NOTE 1 Tables (or functions in statistical computer packages) of the standard normal distribution are readily available which give the proportion of process output expected beyond a particular value of interest, such as a **specification limit** (2.1.4), in terms of standard deviations away from the process mean. This obviates the need to work out the statistical distribution function given in the example.

NOTE 2 The function relates to a theoretical distribution. In practice, with empirical distributions, the parameters are replaced by estimates.

[ISO 3534-2:2006, 2.5.5]

2.1.14
total fraction nonconforming
p_t
sum of **upper fraction nonconforming** (2.1.12) and **lower fraction nonconforming** (2.1.13)

EXAMPLE In a normal **distribution** (2.1.7), with mean, μ, and standard deviation, σ:

$$p_t = \Phi\left(\frac{\mu - U}{\sigma}\right) + \Phi\left(\frac{L - \mu}{\sigma}\right)$$

where

p_t is the total fraction nonconforming;

Φ is the distribution function of the standard normal distribution;

U is the upper specification limit;

L is the lower specification limit.

[ISO 3534-2:2006, 2.5.6]

2.2 Process capability, estimates and indices

2.2.1
process capability
⟨estimate⟩ statistical estimate of the outcome of a characteristic from a process which has been demonstrated to be in a state of statistical control and which describes that process's ability to realize a characteristic that will fulfil the requirements for that characteristic

NOTE 1 The characteristic from the process needs to be documented to have been in statistical control.

NOTE 2 The outcome is a **distribution** (2.1.7), the class of which needs determination and its parameters estimated.

NOTE 3 In certain circumstances, the standard deviation, S_w, that represents only within subgroup variation can be used as an estimator for S_t:

$$S_w \approx \frac{\bar{R}}{d_2} \text{ or } \frac{\sum_{j=1}^{m} S_j}{m c_4} \text{ or } \sqrt{\frac{\sum_{j=1}^{m} S_j^2}{m}}$$

where

$\bar{R}$ is the average range calculated from a set of m subgroup ranges;

S_j is the sample standard deviation of the jth subgroup;

m is the number of subgroups of the same size, n;

d_2, c_4 are constants based on subgroup size, n.

Alternatively, for a normal distribution, the process overall standard deviation σ_t, can be estimated using the formula for S_t.

The value of the estimators S_t and S_w converge for a process in a state of statistical control. A comparison of the two gives an indication of the degree of stability of the process. For an out-of-control process about a constant mean, or for a process that is subject to systematic changes in the mean, the value of S_w is likely to significantly underestimate the process standard deviation. Hence S_w should be used with extreme caution. Sometimes, too, the estimator S_t is preferred to S_w because it has more tractable statistical properties (e.g. facilitating the calculation of confidence limits).

NOTE 4 For a normal distribution, process capability can be assessed from the expression:

$$\overline{\overline{X}} \pm z S_t$$

where

$$\overline{\overline{X}} = \frac{1}{m} \sum_{j=1}^{m} \overline{X}_j$$

and $\overline{X}_j$ is the observed mean of the jth subgroup.

The choice of z depends on the particular parts per million capability standard used. Typically z takes the value of 3, 4 or 5. If the process capability meets the specified requirements, a z value of 3 indicates an expected 2 700 parts per million outside of specification. Similarly, a z of 4 indicates an expected 64 parts per million and a z of 5 an expected 0,6 parts per million outside of specification.

NOTE 5 For a non-normal distribution, process capability can be assessed using, for example, an appropriate probability paper or from the parameters of the distribution fitted to the data. The expression for process capability takes the asymmetric form:

$$\overline{\overline{X}}{}^{+a}_{-b}$$

The notation ${}^{+a}_{-b}$ is in the same style as standard drawing office practice for expressing specified tolerances about a nominal, or preferred, value for a characteristic when the preferred value is not equidistant from each limit. The equivalent notation for limits symmetrical about the preferred value is ±. This enables a direct comparison to be made between the dimensional performance of a characteristic and its specified requirements in terms of both location and dispersion.

NOTE 6 When $S_w = \dfrac{\overline{R}}{d_2}$ is used, it needs to be appreciated that this estimator:

— becomes progressively less efficient as the subgroup size increases;

— is very sensitive to the distribution of individual values;

— makes it difficult to estimate confidence limits.

NOTE 7 Capability conditions are very restrictive and include:

— the methods applied to demonstrate that the process is in control;

— technical conditions (input batches, operators, tools, etc.);

— the measurement process (discrimination, accuracy, repeatability, reproducibility, etc.); and

— data collection (duration, frequency).

NOTE 8 Adapted from ISO 3534-2:2006, 2.7.1.

2.2.2
process capability estimate
quantity that describes one or more properties of the **distribution** (2.1.7) of the product characteristic under capability conditions

EXAMPLE 1 The standard deviation (ISO 3534-1:2006, 2.37) of the **distribution** (2.1.7) of the product characteristic under capability conditions (see 2.2.1, Notes 1 and 7).

EXAMPLE 2 The mean (ISO 3534-1:2006, 2.35.1) of the **distribution** (2.1.7) of the product characteristic under capability conditions (see 2.2.1, Notes 1 and 7).

EXAMPLE 3 The **reference interval** (2.1.11) of the **distribution** (2.1.7) of the product characteristic under capability conditions (see 2.2.1, Notes 1 and 7).

2.2.3
process capability index
C_{p}
index describing **process capability** (2.2.1) in relation to **specified tolerance** (2.1.5)

NOTE 1 Frequently, the process capability index is expressed as the value of the **specified tolerance** (2.1.5) divided by a measure of the length of the **reference interval** (2.1.11) for a process in a state of statistical control, namely as:

$$C_{\mathrm{p}} = \frac{U - L}{X_{99,865\,\%} - X_{0,135\,\%}}$$

NOTE 2 For a normal **distribution** (2.1.7), the **reference interval** (2.1.11) is equal to $6S_{\mathrm{w}}$ (see 2.2.1, Notes).

NOTE 3 For a non-normal **distribution** (2.1.7), the **reference interval** (2.1.11) can be estimated, for example, using the probability paper method or the Pearson curves method.

NOTE 4 Adapted from ISO 3534-2:2006, 2.7.2.

2.2.4
upper process capability index
$C_{\mathrm{pk}U}$
index describing **process capability** (2.2.1) in relation to the upper **specification limit** (2.1.4), U

NOTE 1 Frequently, the upper process capability index is expressed as the difference between the upper **specification limit** (2.1.4) and the 50 % distribution quantile, $X_{50\,\%}$ divided by a measure of the length of the upper **reference interval** (2.1.11) for a process in a state of statistical control, namely as:

$$C_{\mathrm{pk}U} = \frac{U - X_{50\,\%}}{X_{99,865\,\%} - X_{50\,\%}}$$

NOTE 2 For a normal **distribution** (2.1.7), the upper **reference interval** (2.1.11) is equal to $3S_{\mathrm{w}}$ (see 2.2.1, Notes) and $X_{50\,\%}$ represents both the mean and the median.

NOTE 3 For a non-normal **distribution** (2.1.7), the upper **reference interval** (2.1.11) can be estimated, for example, using the probability paper method or the Pearson curves method and $X_{50\,\%}$ represents the median.

NOTE 4 Adapted from ISO 3534-2:2006, 2.7.4.

2.2.5
lower process capability index
$C_{\mathrm{pk}L}$
index describing **process capability** (2.2.1) in relation to the lower **specification limit** (2.1.4)

NOTE 1 Frequently, the lower process capability index is expressed as the difference between the 50 % distribution quantile, $X_{50\,\%}$, and the lower **specification limit** (2.1.4) divided by a measure of the length of the lower **reference interval** (2.1.11) for a process in a state of statistical control, namely as:

$$C_{\mathrm{pk}L} = \frac{X_{50\,\%} - L}{X_{50\,\%} - X_{0,135\,\%}}$$

NOTE 2 For a normal **distribution** (2.1.7) the lower **reference interval** (2.1.11) is equal to $3S_{\mathrm{w}}$ (see 2.2.1, Notes) and $X_{50\,\%}$ represents both the mean and the median.

NOTE 3 For a non-normal **distribution** (2.1.7), the lower **reference interval** (2.1.11) can be estimated, for example, using the probability paper method or the Pearson curves method and $X_{50\,\%}$ represents the median.

NOTE 4 Adapted from ISO 3534-2:2006, 2.7.3.

2.2.6
minimum process capability index
C_{pk}
smaller of **upper process capability index** (2.2.4) and **lower process capability index** (2.2.5)

[ISO 3534-2:2006, 2.7.5]

2.3 Process performance, measures and indices

2.3.1
process performance
⟨measure⟩ statistical measure of the outcome of a characteristic from a process that may not have been demonstrated to be in a state of statistical control

NOTE 1 The characteristic from the process does not need to be documented to have been in statistical control.

NOTE 2 The outcome is a **distribution** (2.1.7) the class of which needs determination and its parameters assessed.

NOTE 3 Care should be exercised in using this measure as it may contain a component of variability due to special causes the value of which is not predictable.

NOTE 4 For a normal **distribution** (2.1.7) described in terms of the standard deviation, S_{t}, estimated from only one sample of size N, the standard deviation is expressed thus:

$$S_{\text{t}} = \sqrt{\frac{\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2}{N-1}}$$

where

$$\bar{X} = \frac{\sum_{i=1}^{N} X_i}{N}$$

The descriptor, S_{t}, takes into account the variation both due to random (common) causes together with any special causes that may be present. S_{t} is used here instead of σ_{t} as the standard deviation is a statistical descriptive measure. The sample size can be made up of m subgroups each of size n.

NOTE 5 For a normal **distribution** (2.1.7) process performance can be assessed from the expression:

$$\bar{X} \pm zS_{\text{t}}$$

The choice of z depends on the particular parts per million performance requirement. Typically z takes the value of 3, 4 or 5. If the process capability meets the specified requirements, a z value of 3 indicates an expected 2 700 parts per million outside of specification. Similarly, a z of 4 indicates an expected 64 parts per million and a z of 5 an expected 0,6 parts per million outside of specification.

NOTE 6 For a non-normal distribution, process performance can be assessed using, for example, an appropriate probability paper or from the parameters of the distribution fitted to the data. The expression for process performance takes the asymmetric form:

$$\bar{X}^{+a}_{-b}$$

The notation ${}^{+a}_{-b}$ is in the same style as standard drawing office practice for expressing specified tolerances about a nominal, or preferred, value for a characteristic when the preferred value is not equidistant from each limit. The equivalent notation for limits symmetrical about the preferred value is ±. This enables a direct comparison to be made between the dimensional performance of a characteristic and its specified requirements in terms of both location and dispersion.

NOTE 7 Performance conditions are least restrictive, but include:

— technical conditions (input batches, operators, tools, etc.);

— the measurement process (discrimination, accuracy, repeatability, reproducibility, etc.); and

— data collection (duration, frequency).

NOTE 8 Adapted from ISO 3534-2:2006, 2.6.1.

2.3.2
process performance measure
quantity that describes one or more properties of the **distribution** (2.1.7) of the product characteristic under performance conditions

EXAMPLE 1 The standard deviation (ISO 3534-1:2006, 2.37) of the **distribution** (2.1.7) of the product characteristic under performance conditions (see 2.3.1, Notes 1 and 7).

EXAMPLE 2 The mean (ISO 3534-1:2006, 2.35.1) of the **distribution** (2.1.7) of the product characteristic under performance conditions (see 2.3.1, Notes 1 and 7).

EXAMPLE 3 The **reference interval** (2.1.11) of the **distribution** (2.1.7) of the product characteristic under performance conditions (see 2.3.1, Notes 1 and 7).

2.3.3
process performance index
P_{p}
index describing **process performance** (2.3.1) in relation to **specified tolerance** (2.1.5)

NOTE 1 Frequently, the process performance index is expressed as the value of the **specified tolerance** (2.1.5) divided by a measure of the length of the **reference interval** (2.1.11) for a process in a state of statistical control, namely as:

$$P_{\mathrm{p}} = \frac{U - L}{X_{99{,}865\,\%} - X_{0{,}135\,\%}}$$

NOTE 2 For a normal **distribution** (2.1.7), the length of the **reference interval** (2.1.11) is equal to $6S_{\mathrm{t}}$ (see 2.3.1, Notes).

NOTE 3 For a non-normal **distribution** (2.1.7), the length of the **reference interval** (2.1.11) can be estimated, for example, using the probability paper method or the Pearson curves method.

NOTE 4 Adapted from ISO 3534-2:2006, 2.6.2.

2.3.4
upper process performance index
$P_{\mathrm{pk}U}$

index describing **process performance** (2.3.1) in relation to the upper **specification limit** (2.1.4), U

NOTE 1 Frequently, the upper process performance index is expressed as the difference between the upper **specification limit** (2.1.4) and the 50 % distribution quantile, $X_{50\,\%}$, divided by a measure of the length of the upper **reference interval** (2.1.11), namely as:

$$P_{\mathrm{pk}U} = \frac{U - X_{50\,\%}}{X_{99{,}865\,\%} - X_{50\,\%}}$$

NOTE 2 For a normal **distribution** (2.1.7), the upper **reference interval** (2.1.11) is equal to $3S_t$ (see 2.3.1, Notes) and $X_{50\,\%}$ represents both the mean and the median.

NOTE 3 For a non-normal **distribution** (2.1.7), the upper **reference interval** (2.1.11) can be estimated, for example, using the probability paper method or the Pearson curves method and $X_{50\,\%}$ represents the median.

NOTE 4 Adapted from ISO 3534-2:2006, 2.6.4.

2.3.5
lower process performance index
$P_{\mathrm{pk}L}$
index describing **process performance** (2.3.1) in relation to the lower **specification limit** (2.1.4), L

NOTE 1 Frequently, the lower process performance index is expressed as the difference between the 50 % distribution quantile, $X_{50\,\%}$, and the lower **specification limit** (2.1.4) divided by a measure of the length of the lower **reference interval** (2.1.11), namely as:

$$P_{\mathrm{pk}L} = \frac{X_{50\,\%} - L}{X_{50\,\%} - X_{0{,}135\,\%}}$$

NOTE 2 For a normal **distribution** (2.1.7), the lower **reference interval** (2.1.11) is equal to $3S_t$ (see 2.3.1, Notes) and $X_{50\,\%}$ represents both the mean and the median.

NOTE 3 For a non-normal **distribution** (2.1.7), the lower **reference interval** (2.1.11) can be estimated, for example, using the probability paper method or the Pearson curves method and $X_{50\,\%}$ represents the median.

NOTE 4 Adapted from ISO 3534-2:2006, 2.5.3.

2.3.6
minimum process performance index
P_{pk}
smaller of **upper process performance index** (2.3.4) and **lower process performance index** (2.3.5)

[ISO 3534-2:2006, 2.6.5]

3 Symbols and abbreviated terms

3.1 Symbols

In addition to the symbols listed below, some symbols are defined where they are used within the text.

α fraction or proportion

β shape parameter in a Weibull distribution

β_2 coefficient of kurtosis

c number of nonconformities in a sample of size n

$\bar{c}$ average number of nonconformities in a sample of size n

c_4 constant based on subgroup size, n (see ISO 8258)

C_{p} process capability index

C_{pk} minimum process capability index

$C_{\mathrm{pk}L}$ lower process capability index

$C_{\text{pk}U}$	upper process capability index
C_{R}	process capability fraction (PCF)
d_2	constant based on subgroup size, n (see ISO 8258)
e	mathematical constant
Φ	distribution function of the standard normal distribution
γ	location parameter in a Weibull distribution
γ_1	coefficient of skewness
m	number of subgroups
K_{l}, K_{u}	multipliers for estimating the confidence limits for a process capability index
L	lower specification limit
$P_{0,135\ \%}$	lower 0,135 % quantile for the Pearson system of curves
μ	location of the process; population mean value
N	total sample size
n	number of values or subgroup size (for a control chart)
np	number nonconforming in a sample of size n
$n\bar{p}$	average number nonconforming in a sample of size n
$P_{\alpha\ \%}$	α percentile
p	proportion of nonconforming items in a sample
$\bar{p}$	average proportion of nonconforming items in a sample
p_L	lower fraction nonconforming
P_{p}	process performance index
P_{pk}	minimum process performance index
$P_{\text{pk}L}$	lower process performance index
$P_{\text{pk}U}$	upper process performance index
p_{t}	total fraction nonconforming
p_U	upper fraction nonconforming
$P_{99,865\ \%}$	upper 99,865 % quantile for the Pearson system of curves
π	geometric constant

Q_k process variation index

θ parameter required for the Rayleigh distribution

$\bar{R}$ average subgroup range

S standard deviation, sample statistic

S_t standard deviation, with the subscript ‘t’ indicating total

$\bar{S}$ average sample standard deviation

S_j observed sample standard deviation of the jth subgroup

σ standard deviation, population

$\hat{\sigma}_t$ estimated standard deviation, total

T target value

U upper specification limit

u number of nonconformities per item in a subgroup

$\bar{u}$ average number of nonconformities per item in a subgroup

$X_{\alpha\,\%}$ $\alpha\,\%$ distribution quantile

X_i ith value in a sample

$\bar{X}$ arithmetic mean value, sample

$\bar{\bar{X}}$ arithmetic mean, of a number of sample arithmetic means

ξ scale parameter in a Weibull distribution

Y_1, Y_2 values read from a graph

z_α quantile of the standardized normal distribution from $-\infty$ to α

3.2 Abbreviated terms

FRC first run capability

MSE mean square error

NHU nonconformities per hundred units

NMU nonconformities per million units

PCF process capability fraction

PCI process capability indices

4 Statistical measures used in process capability and performance

4.1 General

The measures referred to below in 4.2 to 4.6 refer only to measured data. They are unsuitable for count or attribute data and information concerning the expression of measures for such data will be found later in 4.7.

4.2 Measures of location

The most common measure of location used is the mean, μ, although the sample median, $X_{50\,\%}$, is sometimes used. With non-normally distributed process measurements, the median is often a preferred alternative measure.

4.3 Measures of dispersion

4.3.1 Inherent variability

The preferred measure to quantify inherent process variation (ISO 3534-2:2006, 2.2.2) is the standard deviation, σ. This is often estimated from the mean range value, $\overline{R}$, taken from a range (R) chart when the process is stable and in a state of statistical control as indicated in 5.1. Methods used to estimate the process standard deviation are given in Annex A.

4.3.2 Total variability

It is necessary to differentiate between a standard deviation that measures only short-term variation and that which measures longer-term variation. Methods of calculating the standard deviations representing these variations are given in Annex A.

Very often, when data are gathered over a long period of time, the standard deviation is made larger by the effects of fluctuations in the process. When this standard deviation is computed, the symbol $\hat{\sigma}_{\mathrm{t}}$ is used within this part of ISO 22514.

4.4 Mean square error (MSE)

When minimizing variation some practitioners, use the mean square error as the preferred measure. It is compatible with the methods used in off-line quality techniques.

4.5 Reference limits

The lower and upper reference limits are respectively defined as the 0,135 % and the 99,865 % quantiles of the distribution that describe the output of the process characteristic. They are written as $X_{0,135\,\%}$ and $X_{99,865\,\%}$.

4.6 Reference interval, also known as process spread

The reference interval is the interval between the upper and the lower reference limits. The reference interval includes 99,73 % of the individuals in the population from a process that is in a state of statistical control.

4.7 Attributes

Measurement of quality by attributes consists of recording the presence (or absence) of some characteristic or attribute in each of the items in the subgroup under consideration. Counts are made of how many items do (or do not) possess the quality attribute or how many such events occur in the item, group of items or unit area.

When the data are attributive, the usual statistic will be the number of nonconforming items (np) or the number of nonconformities (c) found. Sometimes, it will be necessary to compute the fraction nonconforming (p) or the number of nonconformities per item (u) depending upon the sampling strategies adopted.

5 Capability

5.1 General

Process capability is a measure of inherent process variability. The variability that is inherent in a process when operating in a state of statistical control is known as the inherent process variability. It represents the variation that remains after all known removable *assignable causes* have been eliminated. If the process is monitored using a control chart, the control chart will show an *in control* state.

Capability is often regarded as being related to the proportion of output that will occur within the product specification tolerances. Since a process in statistical control can be described by a predictable distribution, the proportion of out-of-specification outputs can be estimated. As long as the process remains in statistical control, it will continue to produce the same proportion out-of-specification.

Management actions to reduce the variation from *random causes* are required to improve the process' ability to consistently meet the specification requirements.

In short, the following will be necessary:

a) define the process and its operating conditions. If there is a change to those conditions, it will necessitate a new process study;

b) assess the short-term and long-term measurement variabilities as percentages of the total variability and minimize them;

c) preserve the process stability and maintain its statistical control;

d) estimate the remaining inherent variation; and

e) select an appropriate measure of capability.

The following are the conditions that will apply for capability:

— all technical conditions, e.g. temperature and humidity, must be clearly stated;

— the uncertainty of the measurement system must be specified;

— multi-factor, multi-level aspects of the process should be allowed;

— the duration over which the data has been gathered must be recorded;

— the frequency of sampling must be specified and the start and finish dates of data collection;

— the process must be controlled with a control chart; and

— the process must be in a state of statistical control.

It is necessary to check the control chart from which the data have been taken for statistical control and to examine a histogram of the data with any specification limits superimposed upon it. A valid test for normality should be used in assessing the data such as the Anderson-Darling test [8]. This test is powerful in detecting departures from normality in the tails of the distribution and is suggested here as this is the region of interest for capability and performance indices. Additionally, normal probability paper can be used to look for the following:

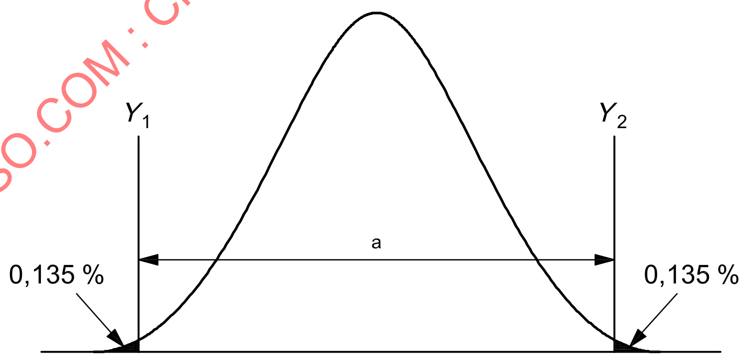
1) verification of normality;
2) outliers;
3) data beyond any specification limit;
4) whether the data are well inside the specification limit(s);
5) evidence of asymmetry (i.e. skewness);
6) evidence of “long tails” in the data (i.e. kurtosis);
7) off-centre distribution; and
8) any unusual features.

Explanations of anomalies should be sought in relation to these mentioned features and appropriate action taken on the data prior to the calculation of any measure. It would be inappropriate to just discard data that do not appear to fit any preconceived pattern. Such departures might be very revealing about the process's behaviour and should be thoroughly investigated.

5.2 Process capability

5.2.1 Normal distribution

Process capability is defined as a statistical measure of inherent process variability for a given characteristic. The conventional method is to take the reference interval which describes 99,73 % of the individual values from a process that is in a state of statistical control with the 0,135 % remaining on each side. This applies even if the population of individual values is not normally distributed. For a normal distribution, this process interval is represented by six standard deviations. See Figure 1.



Key

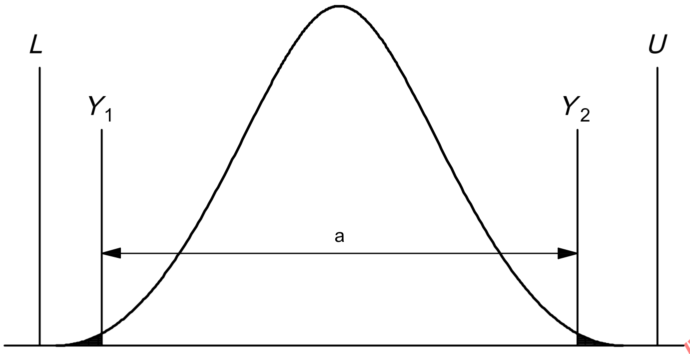
a Reference interval 99,73%.

Figure 1

On occasions, process capability is taken to account for extra sources of variation such as a multiple stream process, for example, output from a multi-cavity injection moulding press. Under these circumstances, the distribution of values may still be approximately normal but with extra variability so that the standard deviation must represent the total variation, σ_t. It is important to state how the standard deviation has been calculated as well as the sampling strategy used, sample size and the quantity and variability of output produced between samples as these will in practice affect the validity of the capability assessment.

Data will usually be taken from a control chart. If the control chart had relaxed control lines or modified control lines, the real process standard deviation will be larger than that estimated from data taken from a control chart with standard control lines. Issues such as these and those given earlier will influence the reference interval and it is important that they are stated in any capability assessment.

“Capable” processes will be those whose reference intervals are less than any specified tolerance by a particular amount. An example of this is shown in Figure 2.



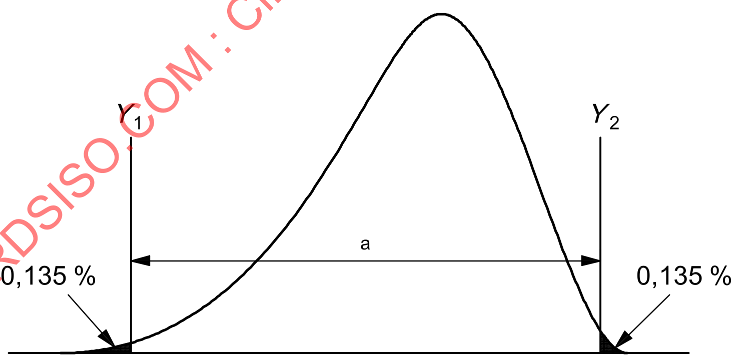
Key

a Reference interval 99,73 %.

Figure 2

5.2.2 Non-normal distribution

If the distribution of individual values does not form a normal distribution but is skewed, then the reference interval may appear as in Figure 3. The values Y_1 and Y_2, which will usually be the 0,135 % and the 99,865 % quantiles, can be estimated using a suitable probability paper (see Figure 4 for an example using an extreme value distribution probability paper) or by the use of suitable computer software (see Annex E). They may also be computed using tabular values (see Annex B) or using the particular probability function as suggested in Annex C.



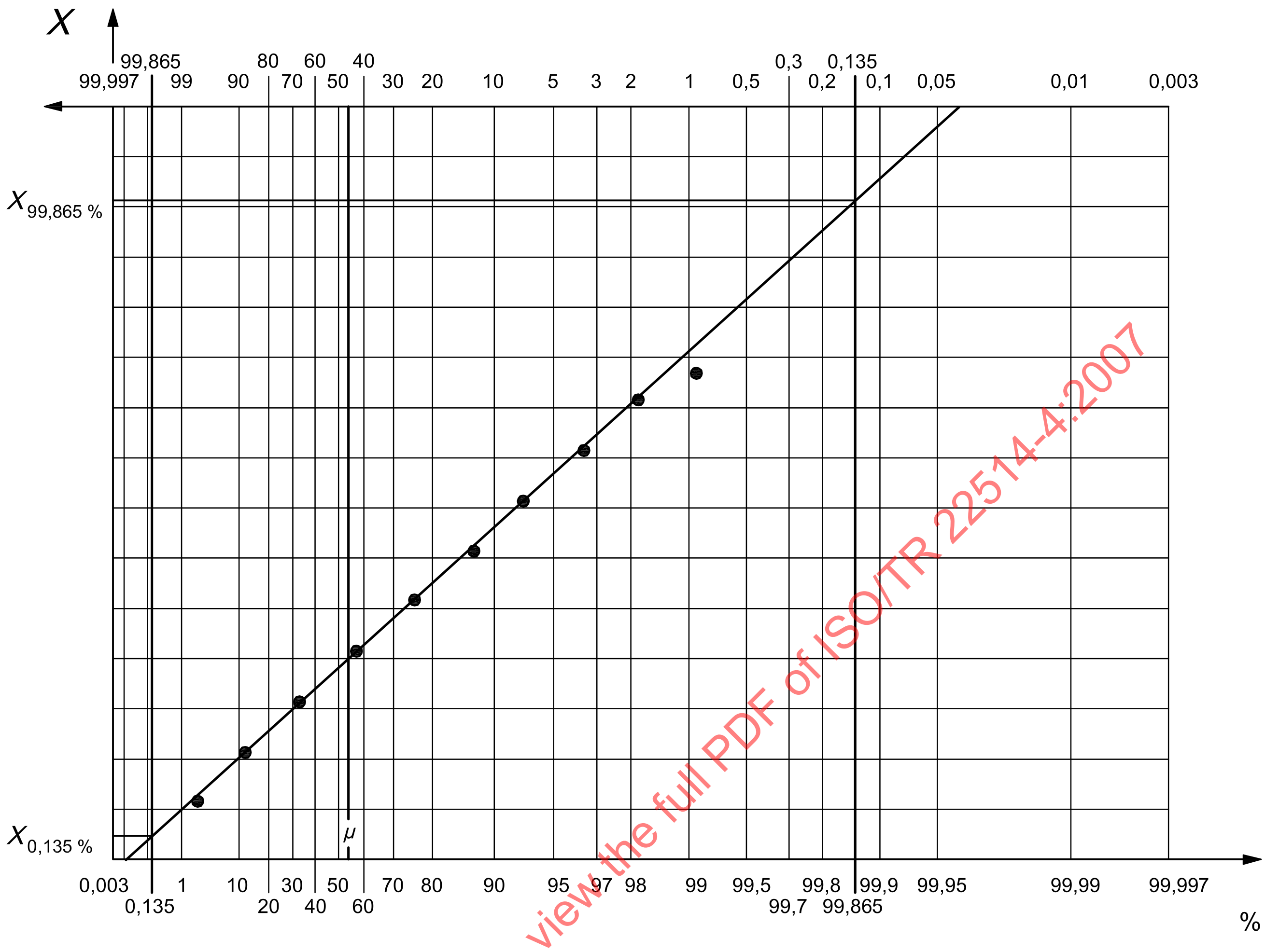
Key

a Reference interval 99,73 %.

Figure 3

5.3 Process location

Even if a process can be deemed capable by the above definition (in 5.2.1), if the process distribution has been poorly centred relative to the specification limits, out-of-specification items may be produced. For this reason, it is necessary to assess the location in addition to the process interval.



Key

——— best fit line

● cumulative percent

Figure 4

5.4 Process capability indices for measured data

5.4.1 General

It should be noted that when the capability indices given in this part of ISO 22514 are computed, they only form point estimates of their true values. It is therefore recommended that wherever possible the indices' confidence intervals are computed and reported. Methods by which these can be computed are described in Annex D.

It is effective to express the capability of a process with the use of an index number. Several indices are given. Care must be taken when handling non-normal distributions.

The process capability index often used is the ratio of a specified tolerance to the reference interval and is known as C_p. Thus:

$$C_p = \frac{U - L}{X_{99,865\%} - X_{0,135\%}} \tag{1}$$

where

L is the lower specification limit;

U is the upper specification limit.

There are other indices that incorporate both the location and the variation; of these, the most widely used is the C_{pk} index. If the index is less than a given value, the process is deemed to be producing an excessive proportion of items outside of the specification.

The C_{pk} index is the ratio of the difference between a specified tolerance limit and the process location to the difference between the corresponding natural process limit and the process location.

$$C_{\text{pk}U} = \frac{U - X_{50\,\%}}{X_{99{,}865\,\%} - X_{50\,\%}} \tag{2}$$

and

$$C_{\text{pk}L} = \frac{X_{50\,\%} - L}{X_{50\,\%} - X_{0{,}135\,\%}}$$

where

L is the lower specification limit;

U is the upper specification limit;

$X_{50\,\%}$ is the median value.

The C_{pk} index is reported as the smaller value of these.

Some practitioners report both of the above values that are also known as CPU and CPL, respectively. This provides information about both sides of the process.

These indices will provide information about whether a process is poorly centred and whether it will possibly produce out-of-specification items. Even if the C_{p} index is high, a low value of the C_{pk} index will reveal a poorly centred process and a high probability of producing out-of-specification items.

5.4.2 The C_{p} index (normal distribution)

If the individual values form a normal distribution, the length of the reference interval is equal to 6σ, where σ is the inherent process standard deviation. Therefore, the C_{p} index can be expressed as:

$$C_{\text{p}} = \frac{U - L}{6\sigma}$$

An estimate ($\hat{\sigma}$) of the inherent process standard deviation (σ) is required to obtain an estimate of the C_{p} index. When this has been obtained, usually with data from a control chart once the process is shown to be statistically stable (see 5.1), the index is estimated:

$$\hat{C}_{\text{p}} = \frac{U - L}{6\hat{\sigma}}$$

5.4.3 The C_{pk} index (normal distribution)

When the distribution of individual values forms a normal distribution, the median $X_{50\ \%}$ is equal to the mean (μ). Further, $X_{99,865\ \%} - X_{50\ \%}$ and $X_{50\ \%} - X_{0,135\ \%}$ are each equal to 3σ. Therefore, the C_{pk} index can be expressed as the minimum of:

$$C_{pkU} = \frac{U - \mu}{3\sigma}$$

or

$$C_{pkL} = \frac{\mu - L}{3\sigma}$$

C_{pk} may be estimated by the minimum of:

$$\hat{C}_{pkU} = \frac{U - \bar{\bar{X}}}{3\hat{\sigma}}$$

or

$$\hat{C}_{pkL} = \frac{\bar{\bar{X}} - L}{3\hat{\sigma}}$$

In computing a capability index, thought must be given to the measure of the process variation used in the denominator. Here, σ is given to represent the variation when the data comes from a process that is in a state of statistical control.

The data might come from a multiple stream process such as a multi-headed filling machine or a multi-spindle machine where the total output is treated together, where data from all streams are simultaneously considered. The lower the index, the greater the proportion of items produced out-of-specification.

5.5 Process capability indices for measured data (non-normal)

5.5.1 General

If the distribution of individual values is non-normal, the expressions in Equations (1) and (2) still apply but the estimation of the indices becomes more complicated. Three approaches to estimate the reference limits are given.

The probability paper method described in 5.5.2 is fairly simple and requires little computation but is somewhat crude. The approach given in 5.5.4 is computationally more involved but is superior to any other method as far as accuracy is concerned.

5.5.2 Probability paper method

From graphs similar to that shown in Figure 4, estimates of the quantiles $X_{0,135\ \%}$ and $X_{99,865\ \%}$ can be obtained. The estimates are denoted by Y_1 and Y_2 respectively and Equation (1) becomes:

$$\hat{C}_p = \frac{U - L}{Y_2 - Y_1}$$

In a similar way, the C_{pk} formulae become:

$$\hat{C}_{\text{pk}U} = \frac{U - X_{50\ \%}}{Y_2 - X_{50\ \%}}$$

or

$$\hat{C}_{\text{pk}L} = \frac{X_{50\ \%} - L}{X_{50\ \%} - Y_1}$$

whichever gives the lower value. If the index is less than a given value, the process is deemed to be producing an excessive proportion of items outside of the specification. The proportion nonconforming depends upon the distribution and the value of the index. The link between the index and the proportion of nonconforming items produced depends on the class of distributions. Care should be taken not to interpret indices on the basis of cut-off points that have been derived for the normal distribution and hence are only applicable for that distribution.

Note that the probability paper method directly estimates fairly extreme quantiles and this can be inaccurate.

5.5.3 Pearson curves method

As an alternative to using probability paper, standardized Pearson curves may be used. The method is described by way of an example (see Annex B). The index is computed using the formula:

$$\hat{C}_{\text{p}} = \frac{U - L}{\hat{X}_{99{,}865\ \%} - \hat{X}_{0{,}135\ \%}}$$

where $\hat{X}_{0{,}135\ \%}$ and $\hat{X}_{99{,}865\ \%}$ are the 0,135 % and 99,865 % quantiles estimated from the standardized Pearson curves.

Also, we have the formulae:

$$\hat{C}_{\text{pk}U} = \frac{U - \hat{X}_{50\ \%}}{\hat{X}_{99{,}865\ \%} - \hat{X}_{50\ \%}}$$

or

$$\hat{C}_{\text{pk}L} = \frac{\hat{X}_{50\ \%} - L}{\hat{X}_{50\ \%} - \hat{X}_{0{,}135\ \%}}$$

where $\hat{X}_{50\ \%}$ is the estimated median.

In order to use this method, it is necessary to establish skewness and kurtosis values in addition to the mean and standard deviation for the data set upon which the index is to be computed.

This method is not preferred but is presented here for completeness due to its occasional use.

This approach, and a similar one based on Johnson curves, should be regarded with considerable caution, especially when it is a procedure within a “black box” computer program used to analyse large sets of data. Some of the potential difficulties are as follows:

— within a system of distributions, some distributions are more difficult to fit than others. The method of moments can yield unstable or inefficient curve parameters in some cases;

— unless the estimation technique is applied skilfully, it is possible to obtain fitted curves that are meaningless over certain ranges of the data. For example, with the method of moments, an easily made mistake is to fit a Pearson Type III distribution whose estimated threshold is less than the lower bound for the process output, thereby invalidating the estimates of $X_{0,135\ \%}$ and C_{pk};

— the method of moments does not yield estimates of the variability in the estimated indices. Likewise, these methods do not yield confidence intervals for the indices;

— not every data distribution can be described adequately with a Pearson or Johnson curve;

— goodness-of-fit tests are limited to the chi-squared test since more powerful tests are not generally available for the Pearson and Johnson systems;

— the “black box” approach tends to displace basic practices, such as plotting the data and applying simple normalizing transformations, that provide genuine understanding of the process.

5.5.4 Distribution identification method

Annex C describes certain families of distribution functions (such as the log-normal distribution, the Rayleigh and the Weibull distributions) that are commonly found when investigating process capability. The method is first to identify the appropriate family of distributions, secondly to estimate the parameters of the distribution of the family that best explain the data by some efficient estimation method and finally to express the quantiles in terms of the parameters of that distribution.

This is analogous to the procedure adopted in the case of the normal distribution where σ is estimated and 6σ is represented by $(X_{99,865\ \%} - X_{0,135\ \%})$.

Various types of probability paper may be useful to identify the appropriate family of distributions.

5.6 Alternative method for describing and calculating process capability estimates

The bases for this method are the widely used definitions of C_{p} and C_{pk} for the “ideal process” with a normally distributed characteristic X, where the expectation μ and variance σ^2 are constant with time and the corresponding estimates are $\bar{\bar{X}}$ and S^2.

Table 1 — Process capability indices and estimates (normal distribution)

Index	Estimate
$C_{\text{p}} = \dfrac{U-L}{6\sigma}$	$\hat{C}_{\text{p}} = \dfrac{U-L}{6S}$
$C_{\text{pk}U} = \dfrac{U-\mu}{3\sigma}$	$\hat{C}_{\text{pk}U} = \dfrac{U-\bar{\bar{X}}}{3S}$
$C_{\text{pk}L} = \dfrac{\mu-L}{3\sigma}$	$\hat{C}_{\text{pk}L} = \dfrac{\bar{\bar{X}}-L}{3S}$
$C_{\text{pk}} = \min(C_{\text{pk}L}, C_{\text{pk}U})$	$\hat{C}_{\text{pk}} = \min(\hat{C}_{\text{pk}L}, \hat{C}_{\text{pk}U})$

This “ideal process” implies that the long-term standard deviation is equal to the short-term standard deviation.

When these measures of process capability have to be extended to characteristics that are not normally distributed, it is necessary to begin from the point of view that these measures are meant to be managerial tools, that they display and reflect the conformance of the actual values of a characteristic with its corresponding specification limits. Therefore, these measures must be linked to the fraction of actual values conforming or nonconforming. In particular, the same fraction conforming or nonconforming has to yield the same values of capability or performance independent of the shape of the distribution of the actual values of a characteristic.

The following formulae are equivalent to those in Table 1:

Table 2 — Process capability indices and estimates (normal distribution) — Equivalent formulae

Index	Estimate
$C_{\mathrm{p}} = \frac{C_{\mathrm{pk}U} + C_{\mathrm{pk}L}}{2}$ $C_{\mathrm{pk}U} = \frac{z_{1-p_U}}{3}$ $C_{\mathrm{pk}L} = \frac{z_{1-p_L}}{3}$	$\hat{C}_{\mathrm{p}} = \frac{\hat{C}_{\mathrm{pk}U} + \hat{C}_{\mathrm{pk}L}}{2}$ $\hat{C}_{\mathrm{pk}U} = \frac{z_{1-\hat{p}_U}}{3}$ $\hat{C}_{\mathrm{pk}L} = \frac{z_{1-\hat{p}_L}}{3}$

where p_U and p_L are the fractions nonconforming at the upper and lower specification limits and $\hat{p}_U$, $\hat{p}_L$ are the corresponding estimates. The formulae in the above table can be applied to any distribution.

It is assumed that the user has knowledge of the shape of the distribution because of what is known about the manufacturing process or by some evaluation of a sample by an appropriate probability paper.

For those distributions that are frequently observed (normal, log-normal, Rayleigh and Weibull), the required relations and formulae are given in Annex C.

5.7 Other capability indices for measured data in other special cases

5.7.1 Process capability fraction (PCF)

The PCF is the inverse of the C_{p} index:

$$\frac{6\sigma}{U - L} = \frac{1}{C_{\mathrm{p}}}$$

It can be expressed as a percentage value and occasionally named C_{R} (%).

5.7.2 Indices when the specification limit is one-sided or no specification limit is given

5.7.2.1 General

Sometimes, a specification is given that has only one limit, e.g. a maximum value. In these circumstances, it will only be possible to compute a C_{pk} or a P_{pk} index.

There will also be situations when specification limits are not given or not known. However, if a target or nominal value is given for the product characteristic or process parameter the following measures might be appropriate. They present a special appeal to those engaged in minimizing process variation around a target value.

5.7.2.2 Mean square error (MSE)

The mean square error provides a measure that involves both location and variation. It is computed as follows:

$$\sigma^2 + (\mu - T)^2$$

where

σ is the standard deviation of the process;

μ is the achieved mean value of the process;

T is the given target value for the process.

In deriving this measure from data, it is necessary to provide estimates of the process standard deviation and μ using sample data from a control chart.

5.7.2.3 Q_{k} index

This index uses the mean square error given in 5.7.2.2, but expresses the whole value as a coefficient of variation and is computed as follows:

$$Q_{\mathrm{k}} = \frac{100\sqrt{\sigma^2 + (\mu - T)^2}}{T} \ (\%)$$

for $T \neq 0$.

An interesting property of this index is if the process drifts from its target, the index will increase in value, and if the process variation increases, it will also increase the value of the index. The smaller this index becomes, the better the process is deemed to have performed.

5.8 Assessment of proportion out-of-specification (normal distribution)

The proportion of out-of-specification items (p_L and p_U) that fall below L and above U can be estimated using properties of the standard normal distribution. Standardized deviates can be calculated as follows:

$$z_{\hat{p}_U} = 3\hat{C}_{\mathrm{pk}U}$$

and

$$z_{\hat{p}_L} = 3\hat{C}_{\mathrm{pk}L}$$

$\hat{p}_U$ and $\hat{p}_L$ are found as the proportions exceeding z_{p_U} and z_{p_L}, respectively, in a standard normal distribution.

Additionally, the process yield can be computed as 100 % minus the total percentage nonconforming, in the case of a controlled process.

If a characteristic, in statistical control and stable, has a $C_{\mathrm{pk}U}$ of 0,86 and a $C_{\mathrm{pk}L}$ of 0,91, the proportion of out-of-specification can be determined using the method given above as follows.

a) Calculate the "upper" standardized deviate:

$$\begin{aligned} z_{\hat{p}_L} &= 3\hat{C}_{\mathrm{pk}L} \\ &= 3 \times 0{,}91 \\ &= 2{,}73 \end{aligned}$$

b) Calculate the "lower" standardized deviate:

$$\begin{aligned} z_{\hat{p}_L} &= 3\hat{C}_{\mathrm{pk}U} \\ &= 3 \times 0{,}86 \\ &= 2{,}58 \end{aligned}$$

c) Using a standard normal table, look up the values $\hat{p}_U$ and $\hat{p}_L$ for the proportions of the distribution beyond the specification limits U and L, z_{p_U} and z_{p_L} respectively.

For convenience and ease of use, Table 3 gives look-up values for the estimated proportion out-of-specification. Table 3 is indexed by $C_{\mathrm{pk}U}$ or $C_{\mathrm{pk}L}$, the process capability indices (PCI). Table 3 should not be used to derive C_p nor C_pk values for attribute data.

Using the above example of a $C_{\mathrm{pk}U}$ of 0,86 and a $C_{\mathrm{pk}L}$ of 0,91, the estimated proportion beyond the specification limits U and L may be read directly from Table 3 as 0,004 9 and 0,003 2.

Table 3 — $C_{\mathrm{pk}U}$ or $C_{\mathrm{pk}L}$ (PCI in the table) and proportion of the normal distribution remaining in the tails of the distribution beyond a specification limit

PCI	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
1,6	$7,9 \times 10^{-07}$	$6,8 \times 10^{-07}$	$5,9 \times 10^{-07}$	$5,0 \times 10^{-07}$	$4,3 \times 10^{-07}$	$3,7 \times 10^{-07}$	$3,2 \times 10^{-07}$	$2,7 \times 10^{-07}$	$2,3 \times 10^{-07}$	$2,0 \times 10^{-07}$
1,5	$3,4 \times 10^{-06}$	$3,0 \times 10^{-06}$	$2,6 \times 10^{-06}$	$2,2 \times 10^{-06}$	$1,9 \times 10^{-06}$	$1,7 \times 10^{-06}$	$1,4 \times 10^{-06}$	$1,2 \times 10^{-06}$	$1,1 \times 10^{-06}$	$9,2 \times 10^{-07}$
1,4	$1,3 \times 10^{-05}$	$1,2 \times 10^{-05}$	$1,0 \times 10^{-05}$	$8,9 \times 10^{-06}$	$7,8 \times 10^{-06}$	$6,8 \times 10^{-06}$	$5,9 \times 10^{-06}$	$5,2 \times 10^{-06}$	$4,5 \times 10^{-06}$	$3,9 \times 10^{-06}$
1,3	$4,8 \times 10^{-05}$	$4,2 \times 10^{-05}$	$3,7 \times 10^{-05}$	$3,3 \times 10^{-05}$	$2,9 \times 10^{-05}$	$2,6 \times 10^{-05}$	$2,3 \times 10^{-05}$	$2,0 \times 10^{-05}$	$1,7 \times 10^{-05}$	$1,5 \times 10^{-05}$
1,2	0,000 2	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1	0,000 1
1,1	0,000 5	0,000 4	0,000 4	0,000 3	0,000 3	0,000 3	0,000 3	0,000 2	0,000 2	0,000 2
1,0	0,001 3	0,001 2	0,001 1	0,001 0	0,000 9	0,000 8	0,000 7	0,000 7	0,000 6	0,000 5
0,9	0,003 5	0,003 2	0,002 9	0,002 6	0,002 4	0,002 2	0,002 0	0,001 8	0,001 6	0,001 5
0,8	0,008 2	0,007 5	0,006 9	0,006 4	0,005 9	0,005 4	0,004 9	0,004 5	0,004 1	0,003 8
0,7	0,017 9	0,016 6	0,015 4	0,014 3	0,013 2	0,012 2	0,011 3	0,010 4	0,009 6	0,008 9
0,6	0,035 9	0,033 6	0,031 4	0,029 4	0,027 4	0,025 6	0,023 9	0,022 2	0,020 7	0,019 2
0,5	0,066 8	0,063 0	0,059 4	0,055 9	0,052 6	0,049 5	0,046 5	0,043 6	0,040 9	0,038 4
0,4	0,115 1	0,109 3	0,103 8	0,098 5	0,093 4	0,088 5	0,083 8	0,079 3	0,074 9	0,070 8
0,3	0,184 1	0,176 2	0,168 5	0,161 1	0,153 9	0,146 9	0,140 1	0,133 5	0,127 1	0,121 0
0,2	0,274 3	0,264 3	0,254 6	0,245 1	0,235 8	0,226 6	0,217 7	0,209 0	0,200 5	0,192 2
0,1	0,382 1	0,370 7	0,359 4	0,348 3	0,337 2	0,326 4	0,315 6	0,305 0	0,294 6	0,284 3
0,0	0,500 0	0,488 0	0,476 1	0,464 1	0,452 2	0,440 4	0,428 6	0,416 8	0,405 2	0,393 6

5.9 Attributes

5.9.1 General

It is often the case with attribute processes that the objective is zero nonconformities or zero nonconforming items. Consequently, process capability in the case of a controlled process is restricted to statements about the level of nonconformities or nonconforming items. This usually takes the form of the process average, for example, $\overline{p}$, and the indices given above and Table 3 do not apply.

If attributes data are generated by a measurement system that evaluates a variables measurement as a pass or fail judgement, care must be taken to minimize any measurement system variation.

5.9.2 Capability measures and indices in the case of nonconforming units (np) or proportion of nonconforming units (p)

When a process is monitored using a np-chart or a p-chart (see ISO 7870-1 for a description), the process capability can be described by the average level $\overline{p}$ or $n\overline{p}$ once the process is shown to be statistically stable.

The process yield, sometimes called the first run capability (FRC), can be computed as shown in Equation (3), i.e. FRC is the percentage of satisfactory items produced:

$$100\left(1-\overline{p}\right) \qquad (\%)$$

or

$$100\left(1-\frac{n\overline{p}}{n}\right) \qquad (\%) \tag{3}$$

5.9.3 Capability measures and indices in the case of number of nonconformities (c) or proportion of nonconformities (u)

When a process is monitored using a c-chart or a u-chart (see ISO 7870-1 for a description), the process performance should be expressed by recording the average level, $\overline{c}$ or $\overline{u}$, once the process is shown to be statistically stable.

Additionally, the rate of occurrence of nonconformities can be computed as the nonconformities per hundred units (NHU):

$$100\left(\frac{\overline{c}}{n}\right)$$

or

$$100\overline{u}$$

where n is the subgroup size taken.

If the occurrence of nonconformities per hundred is so low that the NHU is very much less than the value 1, consider the number of nonconformities per million instead, i.e. NMU. When processing discrete items, this measure is often given as parts per million (ppm).

The NHU is a useful measure when comparing processes under different subgroup sizes and provides information concerning the expected rate of the nonconformity in the output from the process.

6 Performance

6.1 General

Process performance for a characteristic is the achieved distribution of results. The single important difference between *performance* and *capability* is that for performance there is no requirement for the process to be in a state of statistical control nor for the process to be controlled using a control chart. The following are the conditions that will apply for performance:

— all technical conditions, e.g. temperature and humidity, must be clearly stated;

— the uncertainty of the measurement system must be specified;

— multi-factor, multi-level aspects of the process should be allowed;

— the duration over which the data has been gathered must be recorded;

— the frequency of sampling must be specified and the start and finish dates of data collection;

— the process need not be controlled with a control chart;

— the process need not be in a state of statistical control, in particular historical data where the sequence is unknown can be used to evaluate process performance.

Indices are given below to express process performance. Their form is similar to those already given in the clause on capability and the general relationships given in Equations (1) and (2) for measured data are used except they are named P_{p}, $P_{\mathrm{pk}U}$ and $P_{\mathrm{pk}L}$ respectively.

6.2 Process performance indices for measured data (normal distribution)

6.2.1 The P_{p} index

When the individual values form a normal distribution, the length of the reference interval is equal to $6\sigma_{\mathrm{t}}$ where σ_{t} is the total standard deviation. Therefore, the P_{p} index can be expressed as:

$$P_{\mathrm{p}} = \frac{U - L}{6\sigma_{\mathrm{t}}}$$

An estimate, $\hat{\sigma}_{\mathrm{t}}$, of the total standard deviation (σ_{t}) is required to obtain an estimate of the P_{p} index. In practice, $\hat{\sigma}_{\mathrm{t}}$ will be the standard deviation (S_{t}) of all of the data. When this has been obtained, the index is estimated:

6.2.2 The P_{pk} index

When the distribution of individual values forms a normal distribution, the median $X_{50\,\%}$ is equal to the mean (μ). Further, $X_{99{,}865\,\%} - X_{50\,\%}$ and $X_{50\,\%} - X_{0{,}135\,\%}$ are each equal to $3\sigma_{\mathrm{t}}$. Therefore, the P_{pk} index is the smaller of the two values:

$$P_{\mathrm{pk}U} = \frac{U - \mu}{3\sigma_{\mathrm{t}}}$$

or

$$P_{\mathrm{pk}L} = \frac{\mu - L}{3\sigma_{\mathrm{t}}}$$

where P_{pk} is estimated by:

$$\hat{P}_{\mathrm{pk}U} = \frac{U - \overline{X}}{3\hat{\sigma}_{\mathrm{t}}}$$

or

$$\hat{P}_{\mathrm{pk}L} = \frac{\overline{X} - L}{3\hat{\sigma}_{\mathrm{t}}}$$

The lower the index, the greater the proportion of items produced out-of-specification.

6.3 Process performance indices for measured data (non-normal distribution)

6.3.1 General

The approaches adopted in this clause for non-normal data is the same as that given earlier in 5.5 for capability indices.

6.3.2 Probability paper method

From graphs similar to those given in Figure 4, estimates of the quantiles $X_{0,135\,\%}$ and $X_{99,865\,\%}$ can be obtained. The estimates are denoted by Y_1 and Y_2 respectively and the formula becomes:

$$\hat{P}_{\mathrm{p}} = \frac{U - L}{Y_2 - Y_1}$$

In a similar way, the P_{pk} formulae become:

$$\hat{P}_{\mathrm{pk}U} = \frac{U - \hat{X}_{50\,\%}}{Y_2 - \hat{X}_{50\,\%}}$$

or

$$\hat{P}_{\mathrm{pk}L} = \frac{\hat{X}_{50\,\%} - L}{\hat{X}_{50\,\%} - Y_1}$$

whichever gives the lower value. If the index is less than a given value, the process is deemed to have produced an excessive proportion of items outside of the specification. The proportion nonconforming depends upon the distribution and the value of the index. The link between the index and the proportion of nonconforming items produced depends on the class of distributions. Care should be taken not to interpret indices on the basis of cut-off points that have derived for the normal distribution and hence are only applicable for that distribution.

Note that the probability paper method directly estimates fairly extreme quantiles and that this can be inaccurate. Additionally, the estimation method using probability paper, although very simple to use, is nevertheless somewhat crude and computational procedures are preferred (see Annex C).

6.3.3 Pearson curves method

As an alternative to using probability paper, standardized Pearson curves are sometimes used. The method is described by way of an example (see Annex B). The index is computed using the formula:

$$\hat{P}_{\mathrm{p}} = \frac{U - L}{\hat{X}_{99,865\,\%} - \hat{X}_{0,135\,\%}}$$

where $\hat{X}_{0,135\,\%}$ and $\hat{X}_{99,865\,\%}$ are the estimated 0,135 % and 99,865 % quantiles from the standardized Pearson curves.

Also, we have the formulae:

$$\hat{P}_{\mathrm{pk}U} = \frac{U - \hat{X}_{50\,\%}}{\hat{X}_{99,865\,\%} - \hat{X}_{50\,\%}}$$

and

$$\hat{P}_{\text{pk}L} = \frac{\hat{X}_{50\%} - L}{\hat{X}_{50\%} - \hat{X}_{0{,}135\%}}$$

where $\hat{X}_{50\,\%}$ is the estimated median value.

In order to use this method, the user will need to establish skewness and kurtosis values in addition to the mean and standard deviation for the data set upon which the index is to be computed.

This method is not preferred but is presented here due to its occasional use for completeness. See 5.5.3 for further comments about the use of this method.

6.3.4 The distribution identification method

See Annex C for a description of certain families of distribution functions such as the log-normal distribution, the Rayleigh and the Weibull distributions that are commonly found when investigating process performance. See also 5.5.4 for further comments about the method.

6.4 Other performance indices for measured data

All of the indices given earlier for capability will have counterparts when considering performance. Any standard deviation will represent the total variation (σ_{t}) instead of the inherent variation (σ). See 4.3 and 5.2.1.

6.5 Assessment of proportion out-of-specification (normal distribution)

The same method to estimate the proportion of out-of-specification is used here as in 5.8. The reader should substitute $\hat{P}_{\text{pk}U}$ and $\hat{P}_{\text{pk}L}$ respectively for $\hat{C}_{\text{pk}U}$ and $\hat{C}_{\text{pk}L}$. Table 3 may also be used to determine the proportion out-of-specification and the reader should enter the table with $\hat{P}_{\text{pk}U}$ or $\hat{P}_{\text{pk}L}$ instead of $\hat{C}_{\text{pk}U}$ or $\hat{C}_{\text{pk}L}$.

6.6 Attributes

The performance indices will be the same as those given earlier. See 5.9 for a description of the measures.

Annex A
(informative)

Estimating standard deviations

A.1 General

It is necessary to estimate the standard deviation in order to calculate the indices referred to in this part of ISO 22514. There are two types of standard deviation to consider. The first is what might be described as the short-term standard deviation or instantaneous (inherent) standard deviation. It is often calculated from statistics taken from a control chart and this is shown in A.2. The other is the estimation of the total standard deviation and this is described in A.3.

A.2 Inherent standard deviation

A.2.1 Estimation using the mean range value

The inherent (process) standard deviation (the data will be taken from an “in control” control chart) may be estimated from a range control chart using the following formula:

$$\hat{\sigma} = \frac{\overline{R}}{d_2}$$

where d_2 is a control chart factor taken from Table A.1.

Table A.1 — Control chart factors for the estimation of process standard deviation

Subgroup size (n)	d_2	c_4
2	1,128	0,797 9
3	1,693	0,886 2
4	2,059	0,921 3
5	2,326	0,940 0
6	2,534	0,951 5
7	2,704	0,959 4
8	2,847	0,965 0
9	2,970	0,969 3
10 [a]	3,078	0,972 7

[a] Values for d_2 and c_4 may be found in textbooks for sample sizes greater than 10.

A.2.2 Estimation using the mean standard deviation value

If a standard deviation control chart has been used to monitor the within subgroup variation, the inherent (process) standard deviation can be estimated using the following formula:

$$\hat{\sigma} = \frac{\overline{S}}{c_4}$$

where c_4 is a control chart factor taken from Table A.1.

A.2.3 Estimation using subgroup standard deviations

If the within subgroup standard deviation is calculated for every subgroup, the following formula gives a more precise value than those in A.2.1 and A 2.2 for the inherent standard deviation:

$$\hat{\sigma} = \sqrt{\frac{\sum_{j=1}^{m} S_j^{\,2}}{m}}$$

where there are m subgroups of n observations in each.

A.3 Estimation of total standard deviation

When data are generated from a process which is "out-of-control" or if no control chart has been used, it is inappropriate to compute the standard deviation using the methods of A.2. The following formula should be used instead:

$$\hat{\sigma}_{\text{t}} = S_{\text{t}} = \sqrt{\frac{\sum_{i=1}^{N}\left(X_i - \overline{X}\right)^2}{N-1}}$$

The circumstances that lead to the use of this equation are when fluctuations exist in the process mean caused by assignable causes that might not be removable, and this extra variation is to be incorporated with the remaining random cause variation. It is the appropriate measure of variation for use in calculating the performance indices.

When considering multiple stream processes, such as a multiple cavity injection moulding press, it is often desired to treat the data from all the cavities as if they came from a single process. The data from each cavity might form a single normal distribution. However, the reality is often that each cavity produces a slightly different distribution because either the means or the variabilities or both are different. If the data from all of the process streams can be considered to yield a normal distribution, the best estimate of the process variation will be given by this equation.

Annex B
(informative)

Estimating capability and performance measures using Pearson curves — Procedure and example [1)]

B.1 Record specification limits

Upper limit, $U = 0{,}30$

Lower limit, $L = 0{,}20$

B.2 Record process statistics

The process is shown to be statistically "in control"

Mean, $\overline{x} = 0{,}235$

Standard deviation, $\hat{\sigma} = 0{,}012\ 2$

Skewness, $\hat{\gamma}_1 = 0{,}7$ (rounding to one decimal place)

Kurtosis, $\hat{\beta}_2 = 3{,}5$ (rounding to one decimal place)

B.3 Look up standardized 0,135 percentile

For positive skewness, use Table B.1; for negative skewness, use Table B.2.

0,135 percentile, $P_{0{,}135\,\%} = 3{,}056$ by interpolation

B.4 Look up standardized 99,865 percentile

For positive skewness, use Table B.2; for negative skewness, use Table B.1.

99,865 percentile, $P_{99{,}865\,\%} = 4{,}656$ by interpolation

B.5 Look up standardized median in Table B.3

For positive skewness, reverse the sign; for negative skewness, leave positive.

Standardized median, $P_{50\,\%} = -0{,}067\ 5$ by interpolation

---

1) Procedure based on that in Reference [10] (see the Bibliography).

B.6 Compute estimated 0,135 % quantile

$$\begin{aligned}\hat{X}_{0,135\,\%} &= \overline{x} - \hat{\sigma} P_{0,135\,\%} \\ &= 0,235 - (0,012\,2 \times 3,056) \\ &= 0,197\,7\end{aligned}$$

B.7 Compute estimated 99,865 % quantile

$$\begin{aligned}X_{99,865\,\%} &= \overline{x} + \hat{\sigma} P_{99,865\,\%} \\ &= 0,235 + (0,012\,2 \times 4,656) \\ &= 0,291\,8\end{aligned}$$

B.8 Compute estimated median

$$\begin{aligned}\hat{X}_{50\,\%} &= \overline{x} + \hat{\sigma} P_{50\,\%} \\ &= 0,235 + (0,012\,2 \times -0,067\,5) \\ &= 0,234\,2\end{aligned}$$

B.9 Compute process capability indices

$$\begin{aligned}\hat{C}_{\mathrm{p}} &= \frac{U - L}{\hat{X}_{99,865\,\%} - \hat{X}_{0,135\,\%}} \\ &= \frac{0,30 - 0,20}{0,291\,8 - 0,197\,7} \\ &= 1,06\end{aligned}$$

$$\begin{aligned}\hat{C}_{\mathrm{pk}U} &= \frac{U - \hat{X}_{50\,\%}}{\hat{X}_{99,865\,\%} - \hat{X}_{50\,\%}} \\ &= \frac{0,30 - 0,234\,2}{0,291\,8 - 0,234\,2} \\ &= 1,14\end{aligned}$$

$$\begin{aligned}\hat{C}_{\mathrm{pk}L} &= \frac{\hat{X}_{50\,\%} - L}{\hat{X}_{50\,\%} - \hat{X}_{0,135\,\%}} \\ &= \frac{0,234\,2 - 0,20}{0,234\,2 - 0,197\,7} \\ &= 0,94\end{aligned}$$

Table B.1

PEARSON CURVES (STANDARDIZED TAILS)

$P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 \geqslant 0$. $P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 < 0$.

Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
−1,4	1,512	1,421	1,317	1,206	1,092	0,979	0,868	0,762														−1,4
−1,2	1,727	1,619	1,496	1,364	1,230	1,100	0,975	0,858	0,747													−1,2
−1,0	1,966	1,840	1,696	1,541	1,384	1,232	1,089	0,957	0,836													−1,0
−0,8	2,210	2,072	1,912	1,736	1,555	1,377	1,212	1,062	0,927	0,804	0,692											−0,8
−0,6	2,442	2,298	2,129	1,941	1,740	1,539	1,348	1,175	1,023	0,887	0,766	0,656										−0,6
−0,4	2,653	2,506	2,335	2,141	1,930	1,711	1,496	1,299	1,125	0,974	0,841	0,723	0,616									−0,4
−0,2	2,839	2,692	2,522	2,329	2,116	1,887	1,655	1,434	1,235	1,065	0,919	0,791	0,677	0,574								−0,2
0,0	3,000	2,856	2,689	2,500	2,289	2,059	1,817	1,578	1,356	1,163	1,000	0,861	0,739	0,630	0,531							0,0
0,2	3,140	2,986	2,834	2,653	2,447	2,220	1,976	1,726	1,485	1,269	1,086	0,933	0,801	0,686	0,583							0,2
0,4	3,261	3,088	2,952	2,785	2,589	2,368	2,127	1,873	1,619	1,382	1,178	1,008	0,865	0,742	0,634	0,536						0,4
0,6	3,366	3,164	3,045	2,896	2,714	2,502	2,267	2,015	1,754	1,502	1,277	1,087	0,931	0,799	0,658	0,583	0,489					0,6
0,8	3,458	3,222	3,118	2,986	2,821	2,622	2,396	2,148	1,887	1,625	1,381	1,172	1,000	0,857	0,736	0,629	0,533					0,8
1,0	3,539	3,266	3,174	3,058	2,910	2,727	2,512	2,271	2,013	1,748	1,491	1,262	1,072	0,917	0,787	0,675	0,575	0,484				1,0
1,2	3,611	3,300	3,218	3,115	2,983	2,817	2,616	2,385	2,132	1,876	1,602	1,357	1,149	0,979	0,840	0,721	0,617	0,524				1,2
1,4	3,674	3,327	3,254	3,161	3,043	2,893	2,708	2,488	2,243	1,981	1,713	1,456	1,230	1,045	0,894	0,768	0,659	0,562	0,475			1,4
1,6	3,731	3,349	3,282	3,199	3,092	2,957	2,787	2,581	2,345	2,089	1,821	1,556	1,316	1,113	0,950	0,815	0,701	0,600	0,510			1,6
1,8	3,782	3,367	3,306	3,229	3,133	3,011	2,855	2,664	2,438	2,189	1,925	1,664	1,404	1,185	1,008	0,863	0,743	0,638	0,546	0,461		1,8
2,0	3,828	3,382	3,325	3,255	3,167	3,055	2,914	2,736	2,524	2,283	2,023	1,755	1,494	1,261	1,068	0,913	0,785	0,676	0,580	0,494		2,0
2,2	3,870	3,395	3,342	3,277	3,196	3,093	2,964	2,800	2,600	2,369	2,116	1,850	1,584	1,339	1,132	0,964	0,828	0,714	0,615	0,526	0,445	2,2
2,4	3,908	3,405	3,356	3,295	3,220	3,126	3,006	2,855	2,669	2,448	2,202	1,940	1,673	1,420	1,198	1,018	0,873	0,752	0,649	0,557	0,475	2,4
2,6	3,943	3,415	3,367	3,311	3,241	3,153	3,043	2,904	2,730	2,521	2,283	2,026	1,760	1,501	1,267	1,073	0,918	0,791	0,683	0,589	0,504	2,6
2,8	3,975	3,423	3,378	3,324	3,259	3,177	3,075	2,946	2,784	2,586	2,358	2,107	1,844	1,581	1,338	1,131	0,965	0,830	0,717	0,620	0,533	2,8
3,0	4,004	3,430	3,387	3,326	3,274	3,198	3,103	2,983	2,831	2,646	2,427	2,183	1,924	1,661	1,410	1,191	1,013	0,870	0,752	0,651	0,562	3,0
3,2	4,031	3,436	3,395	3,346	3,288	3,216	3,127	3,015	2,874	2,699	2,491	2,254	2,000	1,738	1,483	1,253	1,063	0,911	0,787	0,681	0,590	3,2
3,4	4,056	3,441	3,402	3,356	3,300	3,233	3,149	3,043	2,911	2,747	2,549	2,321	2,072	1,813	1,555	1,317	1,115	0,953	0,822	0,712	0,618	3,4
3,6	4,079	3,446	3,408	3,364	3,311	3,247	3,168	3,069	2,945	2,790	2,602	2,383	2,140	1,884	1,626	1,381	1,169	0,996	0,858	0,744	0,646	3,6
3,8	4,101	3,450	3,414	3,371	3,321	3,259	3,184	3,091	2,974	2,829	2,651	2,440	2,205	1,953	1,695	1,446	1,224	1,041	0,895	0,775	0,674	3,8
4,0	4,121	3,454	3,419	3,378	3,329	3,271	3,200	3,111	3,001	2,864	2,695	2,494	2,265	2,018	1,762	1,510	1,281	1,088	0,932	0,807	0,702	4,0
4,2	4,140	3,458	3,423	3,384	3,337	3,281	3,213	3,129	3,025	2,895	2,735	2,543	2,321	2,080	1,827	1,574	1,338	1,135	0,971	0,839	0,730	4,2
4,4	4,157	3,461	3,428	3,389	3,344	3,290	3,225	3,145	3,047	2,923	2,771	2,588	2,374	2,138	1,889	1,636	1,396	1,184	1,011	0,872	0,758	4,4
4,6	4,174	3,464	3,431	3,394	3,350	3,299	3,236	3,160	3,066	2,949	2,805	2,629	2,424	2,194	1,948	1,697	1,453	1,234	1,052	0,905	0,786	4,6
4,8	4,189	3,466	3,435	3,399	3,356	3,306	3,246	3,173	3,084	2,972	2,835	2,668	2,470	2,246	2,005	1,756	1,510	1,285	1,094	0,939	0,815	4,8
5,0	4,204	3,469	3,438	3,403	3,362	3,313	3,256	3,186	3,100	2,994	2,863	2,703	2,513	2,296	2,059	1,813	1,566	1,336	1,137	0,975	0,844	5,0
5,2	4,218	3,471	3,441	3,406	3,367	3,320	3,264	3,197	3,114	3,013	2,888	2,735	2,562	2,342	2,111	1,867	1,621	1,387	1,181	1,010	0,874	5,2
5,4	4,231	3,473	3,444	3,410	3,371	3,326	3,272	3,207	3,128	3,031	2,911	2,765	2,589	2,386	2,160	1,920	1,675	1,438	1,225	1,047	0,904	5,4

Table B.1 (*continued*)

PEARSON CURVES (STANDARDIZED TAILS)

$P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 \geqslant 0$. $P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 < 0$.

Kurtosis	**Skewness** (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
5,6	4,243	3,475	3,446	3,413	3,375	3,331	3,279	3,216	3,140	3,047	2,933	2,793	2,624	2,427	2,206	1,970	1,727	1,489	1,270	1,085	0,935	5,6
5,8	4,255	3,477	3,448	3,416	3,379	3,336	3,286	3,225	3,152	3,062	2,952	2,818	2,656	2,465	2,260	2,019	1,778	1,539	1,316	1,123	0,966	5,8
6,0	4,266	3,478	3,451	3,419	3,383	3,341	3,292	3,233	3,162	3,076	2,970	2,841	2,685	2,501	2,292	2,065	1,827	1,588	1,361	1,162	0,999	6,0
6,2	4,276	3,480	3,453	3,422	3,386	3,345	3,297	3,240	3,172	3,089	2,987	2,863	2,713	2,535	2,332	2,109	1,874	1,635	1,407	1,202	1,031	6,2
6,4	4,286	3,481	3,454	3,424	3,389	3,349	3,303	3,247	3,181	3,100	3,003	2,883	2,739	2,567	2,369	2,151	1,919	1,682	1,452	1,242	1,065	6,4
6,6	4,296	3,483	3,456	3,426	3,392	3,353	3,308	3,254	3,189	3,111	3,017	2,902	2,763	2,597	2,405	2,191	1,962	1,727	1,496	1,282	1,099	6,6
6,8	4,305	3,484	3,458	3,429	3,395	3,357	3,312	3,260	3,197	3,122	3,030	2,919	2,785	2,624	2,438	2,229	2,004	1,771	1,540	1,323	1,134	6,8
7,0	4,313	3,485	3,459	3,431	3,398	3,360	3,316	3,265	3,204	3,131	3,043	2,936	2,806	2,651	2,469	2,265	2,044	1,814	1,583	1,363	1,169	7,0
7,2	4,322	3,486	3,461	3,432	3,400	3,363	3,321	3,270	3,211	3,140	3,054	2,951	2,825	2,675	2,499	2,300	2,083	1,855	1,625	1,403	1,204	7,2
7,4	4,330	3,487	3,462	3,434	3,403	3,366	3,324	3,275	3,218	3,148	3,065	2,965	2,843	2,698	2,527	2,333	2,120	1,895	1,666	1,443	1,240	7,4
7,6	4,337	3,488	3,464	3,436	3,405	3,369	3,328	3,280	3,224	3,156	3,075	2,978	2,860	2,720	2,554	2,364	2,155	1,933	1,706	1,482	1,276	7,6
7,8	4,344	3,489	3,465	3,437	3,407	3,372	3,331	3,284	3,229	3,164	3,085	2,990	2,876	2,740	2,579	2,394	2,189	1,970	1,744	1,521	1,311	7,8
8,0	4,351	3,490	3,466	3,439	3,409	3,374	3,335	3,289	3,235	3,171	3,094	3,002	2,891	2,759	2,603	2,422	2,221	2,005	1,782	1,559	1,347	8,0
8,2	4,358	3,491	3,467	3,440	3,411	3,377	3,338	3,292	3,240	3,177	3,103	3,013	2,906	2,777	2,625	2,449	2,252	2,040	1,818	1,596	1,382	8,2
8,4	4,365	3,492	3,468	3,442	3,412	3,379	3,340	3,296	3,244	3,183	3,111	3,023	2,919	2,794	2,646	2,475	2,282	2,073	1,854	1,632	1,418	8,4
8,6	4,371	3,492	3,469	3,443	3,414	3,381	3,343	3,300	3,249	3,189	3,118	3,033	2,932	2,810	2,666	2,499	2,310	2,104	1,888	1,667	1,452	8,6
8,8	4,377	3,493	3,470	3,444	3,416	3,383	3,346	3,303	3,253	3,195	3,125	3,042	2,943	2,825	2,685	2,522	2,337	2,135	1,921	1,702	1,486	8,8
9,0	4,382	3,494	3,471	3,445	3,417	3,385	3,348	3,306	3,257	3,200	3,132	3,051	2,955	2,839	2,703	2,544	2,363	2,164	1,953	1,736	1,520	9,0
9,2	4,388	3,495	3,472	3,447	3,418	3,387	3,351	3,309	3,261	3,205	1,138	3,059	2,965	2,853	2,720	2,565	2,388	2,192	1,984	1,768	1,553	9,2
9,4	4,393	3,495	3,473	3,448	3,420	3,388	3,353	3,312	3,265	3,209	3,144	3,067	2,975	2,866	2,738	2,585	2,411	2,219	2,014	1,800	1,586	9,4
9,6	4,398	3,496	3,473	3,449	3,421	3,390	3,355	3,315	3,268	3,214	3,150	3,075	2,985	2,878	2,752	2,604	2,434	2,245	2,042	1,831	1,617	9,6
9,8	4,403	3,496	4,474	3,450	3,422	3,392	3,357	3,317	3,272	3,218	3,156	3,082	2,994	2,890	2,766	2,622	2,456	2,271	2,070	1,861	1,648	9,8
10,0	4,408	3,497	3,475	3,451	3,424	3,393	3,359	3,320	3,275	3,222	3,161	3,088	3,003	2,901	2,780	2,639	2,476	2,295	2,097	1,890	1,679	10,0
10,2					3,425	3,395	3,361	3,322	3,278	3,226	3,166	3,095	3,011	2,911	2,793	2,655	2,496	2,318	2,123	1,918	1,708	10,2
10,4						3,396	3,363	3,325	3,281	3,230	3,171	3,101	3,019	2,921	2,806	2,671	2,515	2,340	2,148	1,945	1,737	10,4
10,6							3,364	3,327	3,283	3,233	3,175	3,107	3,026	2,930	2,818	2,686	2,533	2,361	2,172	1,972	1,765	10,6
10,8								3,329	3,286	3,237	3,179	3,112	3,033	2,940	2,829	2,700	2,551	2,382	2,196	1,998	1,793	10,8
11,0									3,289	3,240	3,184	3,118	3,040	2,948	2,840	2,714	2,567	2,401	2,218	2,023	1,819	11,0
11,2										3,243	3,188	3,123	3,046	2,956	2,851	2,727	2,583	2,420	2,240	2,047	1,845	11,2
11,4											3,191	3,128	3,053	2,964	2,861	2,739	2,598	2,438	2,261	2,070	1,870	11,4
11,6											3,195	3,132	3,058	2,972	2,870	2,751	2,613	2,456	2,281	2,093	1,895	11,6
11,8												3,137	3,064	2,979	2,879	2,762	2,627	2,473	2,301	2,115	1,919	11,8
12,0												3,141	3,070	2,986	2,888	2,773	2,641	2,489	2,320	2,136	1,942	12,0
12,2													3,075	2,993	2,896	2,784	2,653	2,505	2,338	2,157	1,965	12,2

Table B.2

PEARSON CURVES (STANDARDIZED TAILS)

$P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 \geqslant 0$. $P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 < 0$.

Kurtosis	Skewness (γ_1)																					(β_2)
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	
−1,4	1,512	1,584	1,632	1,655	1,653	1,626	1,579	1,516														−1,4
−1,2	1,727	1,813	1,871	1,899	1,895	1,861	1,803	1,726	1,636													−1,2
−1,0	1,966	2,065	2,134	2,170	2,169	2,131	2,061	1,966	1,856													−1,0
−0,8	2,210	2,320	2,400	2,446	2,454	2,422	2,349	2,241	2,108	1,965	1,822											−0,8
−0,6	2,442	2,560	2,648	2,704	2,726	2,708	2,646	2,540	2,395	2,225	2,052	1,885										−0,6
−0,4	2,653	2,774	2,869	2,934	2,969	2,968	2,926	2,837	2,699	2,518	2,314	2,114	1,928									−0,4
−0,2	2,839	2,961	3,060	3,133	3,179	3,194	3,173	3,109	2,993	2,824	2,608	2,373	2,152	1,952								−0,2
0,0	3,000	3,123	3,224	3,303	3,358	3,387	3,385	3,345	3,259	3,116	2,914	2,665	2,405	2,169	1,960							0,0
0,2	3,140	3,261	3,364	3,447	3,510	3,550	3,564	3,546	3,488	3,378	3,206	2,970	2,690	2,412	2,167							0,2
0,4	3,261	3,381	3,484	3,570	3,639	3,688	3,715	3,715	3,681	3,603	3,468	3,264	2,993	2,687	2,398	2,149						0,4
0,6	3,366	3,485	3,588	3,676	3,749	3,805	3,843	3,858	3,844	3,793	3,693	3,529	3,290	2,984	2,658	2,366	2,119					0,6
0,8	3,458	3,575	3,678	3,768	3,844	3,905	3,951	3,978	3,981	3,953	3,883	3,758	3,561	3,283	2,945	2,609	2,322					0,8
1,0	3,539	3,654	3,757	3,847	3,926	3,991	4,044	4,080	4,096	4,087	4,043	3,952	3,797	3,561	3,243	2,881	2,547	2,269				1,0
1,2	3,611	3,724	3,826	3,917	3,997	4,066	4,124	4,167	4,194	4,208	4,177	4,115	3,998	3,808	3,529	3,172	2,798	2,476				1,2
1,4	3,674	3,786	3,887	3,978	4,060	4,131	4,193	4,243	4,278	4,296	4,290	4,252	4,168	4,020	3,789	3,463	3,075	2,705	2,399			1,4
1,6	3,731	3,842	3,942	4,033	4,115	4,189	4,253	4,308	4,351	4,378	4,386	4,367	4,311	4,200	4,015	3,736	3,364	2,961	2,609			1,6
1,8	3,782	3,891	3,990	4,081	4,164	4,239	4,307	4,365	4,414	4,449	4,468	4,472	4,431	4,352	4,209	3,979	3,646	3,238	2,840	2,511		1,8
2,0	3,828	3,936	4,034	4,125	4,208	4,285	4,354	4,416	4,468	4,511	4,539	4,549	4,532	4,479	4,372	4,189	3,907	3,522	3,095	2,719		2,0
2,2	3,870	3,976	4,073	4,164	4,248	4,325	4,396	4,460	4,517	4,564	4,600	4,620	4,619	4,587	4,510	4,369	4,137	3,796	3,370	2,949	2,603	2,2
2,4	3,908	4,013	4,109	4,199	4,283	4,361	4,433	4,500	4,559	4,611	4,653	4,682	4,693	4,678	4,627	4,521	4,336	4,047	3,648	3,201	2,808	2,4
2,6	3,943	4,046	4,142	4,231	4,315	4,394	4,467	4,535	4,597	4,653	4,700	4,736	4,757	4,756	4,725	4,649	4,506	4,269	3,916	3,471	3,033	2,6
2,8	3,975	4,077	4,172	4,261	4,344	4,423	4,498	4,567	4,631	4,690	4,741	4,783	4,812	4,824	4,809	4,758	4,650	4,460	4,160	3,745	3,280	2,8
3,0	4,004	4,105	4,199	4,287	4,371	4,450	4,525	4,596	4,662	4,723	4,777	4,824	4,860	4,882	4,881	4,850	4,771	4,623	4,376	4,007	3,544	3,0
3,2	4,031	4,131	4,224	4,312	4,396	4,475	4,550	4,622	4,689	4,752	4,810	4,861	4,903	4,932	4,944	4,929	4,875	4,762	4,563	4,247	3,813	3,2
3,4	4,056	4,155	4,247	4,335	4,418	4,498	4,573	4,645	4,714	4,779	4,839	4,893	4,940	4,976	4,997	4,996	4,963	4,880	4,723	4,461	4,072	3,4
3,6	4,079	4,177	4,269	4,356	4,439	4,518	4,594	4,667	4,737	4,803	4,865	4,922	4,973	5,015	5,044	5,055	5,038	4,980	4,859	4,647	4,311	3,6
3,8	4,101	4,197	4,288	4,375	4,458	4,537	4,614	4,687	4,757	4,825	4,888	4,948	5,002	5,049	5,085	5,106	5,103	5,066	4,976	4,806	4,524	3,8
4,0	4,121	4,217	4,307	4,393	4,476	4,555	4,631	4,705	4,776	4,845	4,910	4,972	5,029	5,080	5,122	5,150	5,159	5,139	5,075	4,943	4,712	4,0
4,2	4,140	4,234	4,324	4,410	4,492	4,571	4,648	4,722	4,794	4,863	4,929	4,993	5,052	5,107	5,153	5,189	5,208	5,202	5,159	5,059	4,873	4,2
4,4	4,157	4,251	4,340	4,425	4,508	4,587	4,663	4,737	4,809	4,879	4,947	5,012	5,074	5,131	5,181	5,223	5,250	5,257	5,232	5,159	5,012	4,4
4,6	4,174	4,267	4,355	4,440	4,522	4,601	4,677	4,752	4,824	4,895	4,963	5,029	5,093	5,152	5,207	5,253	5,288	5,305	5,295	5,244	5,131	4,6
4,8	4,189	4,281	4,369	4,454	4,535	4,614	4,691	4,765	4,838	4,909	4,978	5,045	5,110	5,172	5,229	5,280	5,321	5,346	5,349	5,318	5,233	4,8
5,0	4,204	4,295	4,383	4,467	4,548	4,627	4,703	4,778	4,851	4,922	4,992	5,060	5,126	5,190	5,249	5,303	5,350	5,383	5,396	5,381	5,320	5,0
5,2	4,218	4,308	4,395	4,479	4,560	4,638	4,715	4,789	4,862	4,934	5,004	5,073	5,141	5,206	5,267	5,325	5,376	5,415	5,437	5,436	5,395	5,2
5,4	4,231	4,321	4,407	4,490	4,571	4,649	4,725	4,800	4,873	4,945	5,016	5,086	5,154	5,220	5,284	5,344	5,399	5,443	5,474	5,483	5,460	5,4

Table B.2 (*continued*)

PEARSON CURVES (STANDARDIZED TAILS)

$P_{99,865\ \%}$ (99,865 percentile) for $\gamma_1 \geqslant 0$. $P_{0,135\ \%}$ (0,135 percentile) for $\gamma_1 < 0$.

Kurtosis	Skewness (γ_1)																					
(β_2)	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	(β_2)
5,6	4,243	4,332	4,418	4,501	4,581	4,659	4,736	4,810	4,884	4,956	5,027	5,097	5,166	5,233	5,299	5,361	5,418	5,468	5,505	5,525	5,516	5,6
5,8	4,255	4,343	4,429	4,511	4,591	4,669	4,745	4,820	4,893	4,966	5,037	5,108	5,177	5,246	5,312	5,376	5,436	5,491	5,533	5,561	5,565	5,8
6,0	4,266	4,354	4,439	4,521	4,600	4,678	4,754	4,829	4,902	4,975	5,046	5,117	5,188	5,257	5,325	5,390	5,452	5,511	5,558	5,593	5,608	6,0
6,2	4,276	4,364	4,448	4,530	4,609	4,695	4,763	4,837	4,911	4,983	5,055	5,126	5,197	5,267	5,336	5,403	5,467	5,529	5,581	5,621	5,645	6,2
6,4	4,286	4,373	4,457	4,538	4,618	4,703	4,771	4,845	4,919	4,991	5,063	5,135	5,206	5,276	5,346	5,414	5,480	5,542	5,600	5,646	5,678	6,4
6,6	4,296	4,382	4,466	4,547	4,626	4,710	4,778	4,853	4,926	4,999	5,071	5,143	5,214	5,285	5,356	5,425	5,492	5,557	5,618	5,669	5,706	6,6
6,8	4,305	4,391	4,474	4,554	4,633	4,717	4,785	4,860	4,933	5,006	5,078	5,150	5,222	5,293	5,364	5,434	5,503	5,569	5,634	5,688	5,732	6,8
7,0	4,313	4,399	4,481	4,562	4,640	4,724	4,792	4,867	4,940	5,013	5,085	5,157	5,229	5,301	5,372	5,443	5,513	5,581	5,648	5,706	5,754	7,0
7,2	4,322	4,406	4,489	4,569	4,647	4,730	4,799	4,873	4,946	5,019	5,091	5,164	5,236	5,308	5,380	5,451	5,522	5,591	5,658	5,722	5,775	7,2
7,4	4,330	4,414	4,496	4,576	4,654	4,736	4,805	4,879	4,952	5,025	5,097	5,170	5,242	5,314	5,387	5,459	5,530	5,601	5,669	5,736	5,792	7,4
7,6	4,337	4,421	4,503	4,582	4,660	4,742	4,811	4,885	4,958	5,031	5,103	5,175	5,248	5,320	5,393	5,466	5,538	5,609	5,679	5,749	5,808	7,6
7,8	4,344	4,428	4,509	4,588	4,666	4,747	4,817	4,890	4,963	5,036	5,109	5,181	5,253	5,326	5,399	5,472	5,545	5,617	5,688	5,760	5,823	7,8
8,0	4,351	4,434	4,515	4,594	4,672	4,753	4,822	4,896	4,969	5,041	5,114	5,186	5,259	5,331	5,404	5,478	5,551	5,624	5,696	5,771	5,836	8,0
8,2	4,358	4,441	4,521	4,600	4,677	4,758	4,827	4,901	4,974	5,046	5,118	5,191	5,263	5,336	5,410	5,483	5,557	5,631	5,704	5,775	5,847	8,2
8,4	4,365	4,447	4,527	4,605	4,682	4,762	4,832	4,905	4,978	5,051	5,123	5,195	5,268	5,341	5,414	5,488	5,562	5,637	5,710	5,783	5,858	8,4
8,6	4,371	4,452	4,532	4,611	4,687	4,767	4,837	4,910	4,983	5,055	5,127	5,200	5,272	5,345	5,419	5,493	5,587	5,642	5,717	5,790	5,867	8,6
8,8	4,377	4,458	4,538	4,616	4,692	4,772	4,841	4,914	4,987	5,059	5,132	5,204	5,276	5,349	5,423	5,497	5,572	5,647	5,722	5,797	5,875	8,8
9,0	4,382	4,463	4,543	4,621	4,697	4,776	4,845	4,918	4,991	5,063	5,135	5,208	5,280	5,353	5,427	5,501	5,576	5,652	5,727	5,803	5,883	9,0
9,2	4,388	4,468	4,548	4,625	4,701	4,780	4,850	4,923	4,995	5,067	5,139	5,211	5,284	5,357	5,431	5,505	5,580	5,656	5,732	5,808	5,890	9,2
9,4	4,393	4,473	4,552	4,630	4,705	4,784	4,854	4,926	4,999	5,071	5,143	5,215	5,287	5,361	5,434	5,509	5,584	5,660	5,736	5,813	5,889	9,4
9,6	4,398	4,478	4,557	4,634	4,710	4,788	4,857	4,930	5,002	5,074	5,146	5,218	5,291	5,364	5,437	5,512	5,587	5,663	5,740	5,817	5,894	9,6
9,8	4,403	4,483	4,561	4,638	4,714	4,791	4,861	4,934	5,006	5,078	5,149	5,222	5,294	5,367	5,440	5,515	5,590	5,667	5,744	5,821	5,898	9,8
10,0	4,408	4,487	4,565	4,642	4,717	4,795	4,865	4,937	5,009	5,081	5,153	5,225	5,297	5,370	5,443	5,518	5,593	5,670	5,747	5,825	5,903	10,0
10,2					4,721	4,798	4,868	4,940	5,012	5,084	5,156	5,228	5,300	5,373	5,446	5,521	5,596	5,673	5,750	5,828	5,906	10,2
10,4							4,871	4,943	5,015	5,087	5,158	5,230	5,303	5,375	5,449	5,523	5,599	5,675	5,753	5,831	5,910	10,4
10,6							4,874	4,947	5,018	5,090	5,161	5,233	5,305	5,378	5,451	5,526	5,601	5,678	5,755	5,834	5,913	10,6
10,8								4,949	5,021	5,092	5,164	5,236	5,308	5,380	5,454	5,528	5,603	5,680	5,757	5,836	5,915	10,8
11,0									5,024	5,095	5,166	5,238	5,310	5,383	5,456	5,530	5,605	5,682	5,760	5,838	5,918	11,0
11,2										5,098	5,169	5,240	5,312	5,385	5,458	5,532	5,607	5,684	5,762	5,840	5,920	11,2
11,4											5,171	5,243	5,314	5,387	5,460	5,534	5,609	5,686	5,763	5,842	5,922	11,4
11,6											5,173	5,245	5,316	5,389	5,462	5,536	5,611	5,687	5,765	5,844	5,924	11,6
11,8												5,247	5,318	5,391	5,464	5,538	5,613	5,689	5,767	5,845	5,925	11,8
12,0												5,249	5,320	5,393	5,465	5,539	5,614	5,690	5,768	5,847	5,927	12,0
12,2													5,322	5,394	5,467	5,541	5,616	5,692	5,769	5,848	5,928	12,2

Table B.3

PEARSON CURVES (STANDARDIZED MEDIAN)

$P_{50\ \%}$ (50 percentile). Change sign for $\gamma_1 > 0$.

Kurtosis (β_2)	Skewness (γ_1) 0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	(β_2)
−1,4	0,000	0,053	0,111	0,184	0,282	0,424	0,627	0,754														−1,4
−1,2	0,000	0,039	0,082	0,132	0,196	0,284	0,412	0,591	0,727													−1,2
−1,0	0,000	0,031	0,065	0,103	0,151	0,212	0,297	0,419	0,586													−1,0
−0,8	0,000	0,026	0,054	0,085	0,123	0,169	0,231	0,317	0,439	0,598	0,681											−0,8
−0,6	0,000	0,023	0,047	0,073	0,104	0,142	0,190	0,254	0,343	0,468	0,616	0,653										−0,6
−0,4	0,000	0,020	0,041	0,064	0,091	0,122	0,161	0,212	0,280	0,375	0,504	0,633	0,616									−0,4
−0,2	0,000	0,018	0,037	0,058	0,081	0,108	0,141	0,183	0,237	0,311	0,413	0,542	0,638	0,574								−0,2
0,0	0,000	0,017	0,034	0,053	0,073	0,097	0,126	0,161	0,206	0,266	0,347	0,456	0,579	0,621	0,531							0,0
0,2	0,000	0,015	0,032	0,049	0,068	0,089	0,114	0,145	0,183	0,233	0,299	0,388	0,501	0,605	0,582							0,2
0,4	0,000	0,014	0,029	0,045	0,063	0,082	0,105	0,132	0,165	0,208	0,263	0,336	0,433	0,545	0,607	0,536						0,4
0,6	0,000	0,013	0,028	0,043	0,059	0,077	0,097	0,122	0,151	0,188	0,235	0,297	0,379	0,481	0,579	0,579	0,489					0,6
0,8	0,000	0,013	0,026	0,040	0,055	0,072	0,091	0,113	0,140	0,172	0,213	0,266	0,336	0,425	0,527	0,590	0,533					0,8
1,0	0,000	0,012	0,025	0,038	0,053	0,068	0,086	0,106	0,130	0,159	0,196	0,242	0,301	0,379	0,474	0,563	0,569	0,484				1,0
1,2	0,000	0,011	0,024	0,036	0,050	0,065	0,082	0,100	0,122	0,148	0,181	0,222	0,274	0,341	0,426	0,520	0,576	0,524				1,2
1,4	0,000	0,011	0,023	0,035	0,048	0,062	0,078	0,095	0,116	0,140	0,169	0,206	0,252	0,310	0,385	0,474	0,554	0,555	0,475			1,4
1,6	0,000	0,010	0,022	0,034	0,046	0,060	0,074	0,091	0,110	0,132	0,159	0,192	0,233	0,285	0,351	0,432	0,518	0,564	0,510			1,6
1,8	0,000	0,010	0,021	0,032	0,044	0,057	0,072	0,087	0,105	0,126	0,151	0,180	0,217	0,264	0,323	0,396	0,480	0,549	0,540	0,461		1,8
2,0	0,000	0,009	0,020	0,031	0,043	0,055	0,069	0,084	0,101	0,120	0,143	0,171	0,204	0,246	0,299	0,365	0,443	0,521	0,552	0,494		2,0
2,2	0,000	0,009	0,020	0,030	0,042	0,054	0,067	0,081	0,097	0,115	0,137	0,162	0,193	0,231	0,279	0,338	0,410	0,488	0,544	0,522	0,445	2,2
2,4	0,000	0,009	0,019	0,029	0,040	0,052	0,065	0,078	0,094	0,111	0,131	0,155	0,183	0,218	0,261	0,315	0,381	0,456	0,524	0,538	0,475	2,4
2,6	0,000	0,008	0,018	0,029	0,039	0,051	0,063	0,076	0,091	0,107	0,126	0,148	0,175	0,207	0,246	0,295	0,355	0,426	0,498	0,539	0,503	2,6
2,8	0,000	0,008	0,018	0,028	0,038	0,049	0,061	0,074	0,088	0,104	0,122	0,143	0,167	0,197	0,233	0,278	0,333	0,398	0,470	0,526	0,522	2,8
3,0	0,000	0,008	0,017	0,027	0,037	0,048	0,059	0,072	0,085	0,101	0,118	0,138	0,161	0,189	0,222	0,263	0,313	0,374	0,443	0,506	0,530	3,0
3,2	0,000	0,008	0,017	0,027	0,037	0,047	0,058	0,070	0,083	0,098	0,114	0,133	0,155	0,181	0,212	0,250	0,296	0,352	0,417	0,483	0,525	3,2
3,4	0,000	0,008	0,017	0,026	0,036	0,046	0,057	0,068	0,081	0,095	0,111	0,129	0,150	0,174	0,203	0,239	0,281	0,333	0,394	0,460	0,513	3,4
3,6	0,000	0,007	0,016	0,025	0,035	0,045	0,056	0,067	0,079	0,093	0,108	0,125	0,145	0,168	0,196	0,228	0,268	0,316	0,373	0,437	0,495	3,6
3,8	0,000	0,007	0,016	0,025	0,034	0,044	0,054	0,066	0,078	0,091	0,105	0,122	0,141	0,163	0,188	0,219	0,256	0,301	0,354	0,415	0,475	3,8
4,0	0,000	0,007	0,015	0,025	0,034	0,043	0,053	0,064	0,076	0,089	0,103	0,119	0,137	0,158	0,182	0,211	0,246	0,288	0,337	0,395	0,455	4,0
4,2	0,000	0,007	0,015	0,024	0,033	0,043	0,053	0,063	0,075	0,087	0,101	0,116	0,133	0,153	0,176	0,204	0,236	0,276	0,322	0,376	0,435	4,2
4,4	0,000	0,007	0,015	0,024	0,033	0,042	0,052	0,062	0,073	0,085	0,099	0,113	0,130	0,149	0,171	0,197	0,228	0,265	0,308	0,359	0,416	4,4
4,6	0,000	0,007	0,015	0,023	0,032	0,041	0,051	0,061	0,072	0,084	0,097	0,111	0,127	0,145	0,167	0,191	0,220	0,255	0,296	0,344	0,399	4,6
4,8	0,000	0,006	0,015	0,023	0,032	0,041	0,050	0,060	0,071	0,082	0,095	0,109	0,124	0,142	0,162	0,186	0,213	0,246	0,285	0,330	0,382	4,8
5,0	0,000	0,006	0,014	0,023	0,031	0,040	0,049	0,059	0,070	0,081	0,093	0,107	0,122	0,139	0,158	0,181	0,207	0,238	0,274	0,317	0,367	5,0
5,2	0,000	0,006	0,014	0,022	0,031	0,040	0,049	0,058	0,069	0,080	0,092	0,105	0,119	0,136	0,155	0,176	0,201	0,231	0,265	0,306	0,353	5,2
5,4	0,000	0,006	0,014	0,022	0,030	0,039	0,048	0,057	0,068	0,078	0,090	0,103	0,117	0,133	0,151	0,172	0,196	0,224	0,257	0,295	0,340	5,4

Table B.3 (*continued*)

PEARSON CURVES (STANDARDIZED MEDIAN)

$P_{50\,\%}$ (50 percentile). Change sign for $\gamma_1 > 0$.

Kurtosis (β_2)	Skewness (γ_1) 0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0	(β_2)
5,6	0,000	0,006	0,014	0,022	0,030	0,039	0,047	0,057	0,067	0,077	0,089	0,101	0,115	0,131	0,148	0,168	0,191	0,218	0,249	0,285	0,328	5,6
5,8	0,000	0,006	0,014	0,022	0,030	0,038	0,047	0,056	0,066	0,076	0,087	0,100	0,113	0,128	0,145	0,164	0,186	0,212	0,242	0,277	0,317	5,8
6,0	0,000	0,006	0,014	0,021	0,029	0,038	0,046	0,055	0,065	0,075	0,086	0,098	0,111	0,126	0,142	0,161	0,182	0,207	0,235	0,268	0,307	6,0
6,2	0,000	0,006	0,013	0,021	0,029	0,037	0,046	0,055	0,064	0,074	0,085	0,097	0,110	0,124	0,140	0,158	0,178	0,202	0,229	0,261	0,298	6,2
6,4	0,000	0,006	0,013	0,021	0,029	0,037	0,045	0,054	0,063	0,073	0,084	0,096	0,108	0,122	0,137	0,155	0,175	0,197	0,223	0,254	0,289	6,4
6,6	0,000	0,006	0,013	0,021	0,028	0,037	0,045	0,054	0,063	0,073	0,083	0,094	0,107	0,120	0,135	0,152	0,171	0,193	0,218	0,247	0,281	6,6
6,8	0,000	0,006	0,013	0,021	0,028	0,036	0,044	0,053	0,062	0,072	0,082	0,093	0,105	0,118	0,133	0,150	0,168	0,189	0,213	0,241	0,273	6,8
7,0	0,000	0,005	0,013	0,020	0,028	0,036	0,044	0,053	0,061	0,071	0,081	0,092	0,104	0,117	0,131	0,147	0,165	0,185	0,209	0,236	0,267	7,0
7,2	0,000	0,005	0,013	0,020	0,028	0,036	0,044	0,052	0,061	0,070	0,080	0,091	0,103	0,115	0,129	0,145	0,162	0,182	0,205	0,230	0,260	7,2
7,4	0,000	0,005	0,013	0,020	0,027	0,035	0,043	0,052	0,060	0,070	0,079	0,090	0,101	0,114	0,128	0,143	0,160	0,179	0,201	0,226	0,254	7,4
7,6	0,000	0,005	0,012	0,020	0,027	0,035	0,043	0,051	0,060	0,069	0,079	0,089	0,100	0,113	0,126	0,141	0,157	0,176	0,197	0,221	0,249	7,6
7,8	0,000	0,005	0,012	0,020	0,027	0,035	0,043	0,051	0,059	0,068	0,078	0,088	0,099	0,111	0,124	0,139	0,155	0,173	0,193	0,217	0,243	7,8
8,0	0,000	0,005	0,012	0,019	0,027	0,034	0,042	0,050	0,059	0,068	0,077	0,087	0,098	0,110	0,123	0,137	0,153	0,170	0,190	0,213	0,238	8,0
8,2	0,000	0,005	0,012	0,019	0,027	0,034	0,042	0,050	0,058	0,067	0,076	0,086	0,097	0,109	0,121	0,135	0,151	0,168	0,187	0,209	0,234	8,2
8,4	0,000	0,005	0,012	0,019	0,026	0,034	0,042	0,050	0,058	0,067	0,076	0,086	0,096	0,108	0,120	0,134	0,149	0,165	0,184	0,205	0,229	8,4
8,6	0,000	0,005	0,012	0,019	0,026	0,034	0,041	0,049	0,057	0,066	0,075	0,085	0,095	0,107	0,119	0,132	0,147	0,163	0,181	0,202	0,225	8,6
8,8	0,000	0,005	0,012	0,019	0,026	0,033	0,041	0,049	0,057	0,066	0,075	0,084	0,094	0,106	0,118	0,131	0,145	0,161	0,179	0,199	0,221	8,8
9,0	0,000	0,005	0,012	0,019	0,026	0,033	0,041	0,049	0,057	0,065	0,074	0,084	0,094	0,105	0,116	0,129	0,143	0,159	0,176	0,196	0,218	9,0
9,2	0,000	0,005	0,012	0,019	0,026	0,033	0,040	0,048	0,056	0,065	0,073	0,083	0,093	0,104	0,115	0,128	0,142	0,157	0,174	0,193	0,214	9,2
9,4	0,000	0,005	0,012	0,019	0,026	0,033	0,040	0,048	0,056	0,064	0,073	0,082	0,092	0,103	0,114	0,127	0,140	0,155	0,172	0,190	0,211	9,4
9,6	0,000	0,005	0,012	0,019	0,025	0,033	0,040	0,048	0,055	0,064	0,072	0,082	0,091	0,102	0,113	0,125	0,139	0,153	0,170	0,188	0,208	9,6
9,8	0,000	0,005	0,012	0,018	0,025	0,032	0,040	0,047	0,055	0,063	0,072	0,081	0,091	0,101	0,112	0,124	0,137	0,152	0,168	0,185	0,205	9,8
10,0	0,000	0,005	0,011	0,018	0,025	0,032	0,040	0,047	0,055	0,063	0,071	0,080	0,090	0,100	0,111	0,123	0,136	0,150	0,166	0,183	0,202	10,0
10,2	0,000					0,032	0,039	0,047	0,054	0,063	0,071	0,080	0,089	0,099	0,110	0,122	0,135	0,149	0,164	0,181	0,200	10,2
10,4	0,000					0,032	0,039	0,047	0,054	0,062	0,071	0,079	0,089	0,099	0,109	0,121	0,133	0,147	0,162	0,179	0,197	10,4
10,6	0,000						0,039	0,046	0,054	0,062	0,070	0,079	0,088	0,098	0,109	0,120	0,132	0,146	0,160	0,177	0,195	10,6
10,8	0,000							0,046	0,054	0,061	0,070	0,078	0,088	0,097	0,108	0,119	0,131	0,144	0,159	0,175	0,192	10,8
11,0	0,000								0,053	0,061	0,069	0,078	0,087	0,097	0,107	0,118	0,130	0,143	0,157	0,173	0,190	11,0
11,2	0,000									0,061	0,069	0,078	0,087	0,096	0,106	0,117	0,129	0,142	0,156	0,171	0,188	11,2
11,4	0,000										0,069	0,077	0,086	0,095	0,105	0,116	0,128	0,141	0,154	0,169	0,186	11,4
11,6	0,000										0,068	0,077	0,086	0,095	0,104	0,116	0,127	0,139	0,153	0,168	0,184	11,6
11,8	0,000											0,076	0,085	0,094	0,104	0,115	0,126	0,138	0,152	0,166	0,182	11,8
12,0	0,000											0,076	0,085	0,094	0,104	0,114	0,125	0,137	0,150	0,165	0,181	12,0
12,2	0,000												0,084	0,093	0,103	0,113	0,124	0,136	0,149	0,163	0,179	12,2