TECHNICAL SPECIFICATION

ISO/TS 14253-2

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1999-12-01

Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment —

Part 2:
Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification

Spécification géométrique des produits (GPS) — Vérification par la mesure des pièces et des équipements de mesure —

Partie 2: Guide pour l'estimation de l'incertitude dans les mesures GPS, dans l'étalonnage des équipements de mesure et dans la vérification des produits

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In other circumstances, particularly when there is an urgent market requirement for such documents, a technical committee may decide to publish other types of normative document:

— an ISO Publicly Available Specification (ISO/PAS) represents an agreement between technical experts in an ISO working group and is accepted for publication if it is approved by more than 50 % of the members of the parent committee casting a vote;

— an ISO Technical Specification (ISO/TS) represents an agreement between the members of a technical committee and is accepted for publication if it is approved by 2/3 of the members of the committee casting a vote.

An ISO/PAS or ISO/TS is reviewed every three years with a view to deciding whether it can be transformed into an International Standard.

Attention is drawn to the possibility that some of the elements of this Technical Specification may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TS 14253-2 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO 14253 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment*:

— *Part 1: Decision rules for proving conformance or non-conformance with specification*

— *Part 2: Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification* [Technical Specification]

— *Part 3: Procedures for evaluating the integrity of uncertainty in measurement values*

Annexes A to D of this Technical Specification are for information only.

Introduction

This Technical Specification is a global GPS technical report (see ISO/TR 14638:1995). This global GPS Technical Report influences chain link 4, 5 and 6 in all chains of standards.

For more detailed information of the relation of this report to other standards and the GPS matrix model, see annex D.

This Technical Specification is developed to support ISO 14253-1. This Technical Specification establishes a simplified, iterative procedure of the concept and the way to evaluate and determine uncertainty (standard uncertainty and expanded uncertainty) of measurement, and the recommendations of the format to document and report the uncertainty of measurement information as given in *"Guide to the expression of uncertainty in measurement"* (GUM). In most cases only very limited resources are necessary to estimate uncertainty of measurement by this simplified, iterative procedure, but the procedure may lead to a slight overestimation of the uncertainty of measurement. If a more accurate estimation of the uncertainty of measurement is needed, the more elaborated procedures of the GUM must be applied.

This simplified, iterative procedure of the GUM methods is intended for GPS measurements, but may be used in other areas of industrial (applied) metrology.

Uncertainty of measurement and the concept of handling uncertainty of measurement being of importance to all the technical functions in a company, this Technical Specification relates to e.g. management function, design and development function, manufacture function, quality assurance function, metrology function, etc.

This Technical Specification is of special importance in relation to ISO 9000 quality assurance systems, where it is a requirement that the uncertainty of measurement is known [e.g. 4.11.1, 4.11.2 a) and 4.11.2 b) of ISO 9001:1994].

In this Technical Specification the uncertainty of the result of a process of calibration and a process of measurement is handled in the same way:

— calibration is treated as "measurement of metrological characteristics of a measuring equipment or a measurement standard";

— measurement is treated as "measurement of geometrical characteristics of a workpiece".

Therefore, in most cases no distinction is made in the text between measurement and calibration. The term "measurement" is used as a synonym for both.

Geometrical product specifications (GPS) — Inspection by measurement of workpieces and measuring equipment —

Part 2:
Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification

1 Scope

This Technical Specification gives guidance on the implementation of the concept of *"Guide to the estimation of uncertainty in measurement"* (in short GUM) to be applied in industry for the calibration of (measurement) standards and measuring equipment in the field of GPS and the measurement of workpiece GPS-characteristics. The aim is to promote full information on how to achieve uncertainty statements and provide the basis for international comparison of results of measurements and their uncertainties (relationship between purchaser and supplier).

This Technical Specification is intended to support ISO 14253-1. This Technical Specification and ISO 14253-1 are beneficial to all technical functions in a company in the interpretation of GPS specifications (i.e. tolerances of workpiece characteristics and values of maximum permissible errors (MPE) for metrological characteristics of measuring equipment).

This Technical Specification introduces the Procedure for Uncertainty MAnagement (PUMA), which is a practical, iterative procedure based on the GUM for estimating uncertainty of measurement without changing the basic concepts of the GUM and is intended to be used generally for estimating uncertainty of measurement and giving statements of uncertainty for:

— single results of measurement;

— comparison of two or more results of measurement;

— comparison of results of measurement — from one or more workpieces or pieces of measurement equipment — with given specifications [i.e. maximum permissible errors (MPE) for a metrological characteristic of a measurement instrument or measurement standard, and tolerance limits for a workpiece characteristic, etc.], for proving conformance or non-conformance with the specification.

The iterative method is based basically on an upper bound strategy, i.e. overestimation of the uncertainty at all levels, but the iterations control the amount of overestimation. Intentional overestimation — and not under-estimation — is necessary to prevent wrong decisions based on measurement results. The amount of overestimation shall be controlled by economical evaluation of the situation.

The iterative method is a tool to maximize profit and minimize cost in the metrological activities of a company. The iterative method/procedure is economically self-adjusting and is also a tool to change/reduce existing uncertainty in measurement with the aim of reducing cost in metrology (manufacture). The iterative method makes it possible to compromise between risk, effort and cost in uncertainty estimation and budgeting.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this Technical Specification. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this Technical Specification are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 1:1975, *Standard reference temperature for industrial length measurements*.

ISO 4288:1996, *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Rules and procedures for the assessment of surface texture*.

ISO 9001:1994, *Quality systems — Model for quality systems in design, development, production, installation and servicing*.

ISO 9004-1:1994, *Quality management and quality system elements — Part 1: Guidelines*.

ISO 14253-1:1998, *Geometrical Product Specification (GPS) — Inspection by measurement of workpieces and measuring instruments — Part 1: Decision rules for proving conformance or non-conformance with specifications*.

ISO 14253-3:—[1)], *Geometrical Product Specification (GPS) — Inspection by measurement of workpieces and measuring instruments — Part 3: Procedures for evaluating the integrity of uncertainty of measurement values.*

ISO 14660-1:1999, *Geometrical Product Specification (GPS) — Geometric features — Part 1: General terms and definitions*.

Guide to the expression of uncertainty in measurement (GUM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1st edition, 1995.

International Vocabulary of Basic and General Terms in Metrology (VIM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 2nd edition, 1993.

3 Terms and definitions

For the purposes of this Technical Specification, the terms and definitions given in ISO 14253-1, ISO 14660-1, VIM, GUM and the following apply.

3.1
black box model for uncertainty estimation
method of/model for uncertainty estimation in which the output value of a measurement is obtained in the same unit as the input (stimuli), rather than by measurement of other quantities functionally related to the measurand

NOTE 1 In the black box model — in this Technical Specification — the uncertainty components are assumed additive, the influence quantities is transformed to the unit of the measurand and the sensitivity coefficients are equal to 1.

NOTE 2 In many cases a complex method of measurement may be looked upon as one simple black box with stimulus in and result out from the black box. When a black box is opened, it may turn out to contain several "smaller" black boxes and/or several transparent boxes.

NOTE 3 The method of uncertainty estimation remains a black box method even if it is necessary to make supplementary measurements to determine the values of influence quantities in order to make corresponding corrections.

1) To be published.

3.2
transparent box model for uncertainty estimation
method of/model for uncertainty estimation in which the value of a measurand is obtained by measurement of other quantities functionally related to the measurand

3.3
measuring task
quantification of a measurand according to its definition

3.4
basic measurement task (basic measurement)
measurement task(s) which form the basis for evaluation of more complicated characteristics of a workpiece or a measuring equipment

NOTE Examples of a basic measurement are:

a) one of several individual measurements of the deviation from straightness of a feature of a workpiece;

b) one of the individual measurements of error of indication of a micrometer when measuring the range of error of indication.

3.5
overall measurement task
complicated measuring task, which is evaluated on the basis of several and maybe different basic measurements

NOTE Examples of an overall measuring task are:

a) the measurement of straightness of a feature of a workpiece;

b) the range of error of indication of a micrometer.

3.6
expanded uncertainty (of a measurement)
U
[3.16 of ISO 14253-1:1998 and 2.3.5 of GUM:1995]

NOTE U (capital) always indicates expanded uncertainty of measurement.

3.7
true uncertainty
U_A
uncertainty of measurement that would be obtained by a perfect uncertainty estimation

NOTE 1 True uncertainties are by nature indeterminate.

NOTE 2 See also 8.8.

3.8
conventional true uncertainty — GUM uncertainty
U_c
uncertainty of measurement estimated completely according to the more elaborate procedures of GUM

NOTE 1 The conventional true uncertainty of measurement may differ from an uncertainty of measurement estimated according to this Technical Specification.

NOTE 2 See also 8.8.

3.9
approximated uncertainty
U_{EN}
uncertainty of measurement estimated by the simplified, iterative method

NOTE 1 The index N indicates that U_{EN} is assessed by iteration number N. The designation U_E may be used without indication of the iteration number, when it is without importance to know the number of iterations.

NOTE 2 See also 8.8.

3.10
target uncertainty (for a measurement or calibration)
U_T
uncertainty determined as the optimum for the measuring task

NOTE 1 Target uncertainty is the result of a management decision involving e.g. design, manufacturing, quality assurance, service, marketing, sales and distribution.

NOTE 2 Target uncertainty is determined (optimized) taking into account the specification [tolerance or maximum permissible error (MPE)], the process capability, cost, criticality and the requirements of 4.11.1, 4.11.2 of ISO 9001:1994, 13.1 of ISO 9004-1:1994 and ISO 14253-1.

NOTE 3 See also 8.8.

3.11
required uncertainty of measurement
U_R
uncertainty required for a given measurement process and task

NOTE See also 6.2. The required uncertainty may be specified by, for example, a customer.

3.12
uncertainty management
process of deriving an adequate measurement procedure from the measuring task and the target uncertainty by using uncertainty budgeting techniques

3.13
uncertainty budget (for a measurement or calibration)
statement summarizing the estimation of the uncertainty components that contributes to the uncertainty of a result of a measurement

NOTE 1 The uncertainty of the result of the measurement is unambiguous only when the measurement procedure (including the measurement object, measurand, measurement method and conditions) is defined.

NOTE 2 The term "budget" is used for the assignment of numerical values to the uncertainty components, their combination and expansion, based on the measurement procedure, measurement conditions and assumptions.

3.14
uncertainty contributor
xx
source of uncertainty of measurement for a measuring process

3.15
limit value (variation limit) for an uncertainty contributor
a_{xx}
absolute value of the extreme value(s) of the uncertainty contributor, xx

3.16
uncertainty component
u_{xx}
standard uncertainty of the uncertainty contributor, xx

NOTE The iteration method uses the designation u_{xx} for all uncertainty components. This is not consistent with the present version of GUM which sometimes uses the designation s_{xx} for uncertainty components evaluated by A evaluation and the designation u_{xx} for uncertainty components evaluated by B evaluation.

3.17
influence quantity of a measurement instrument
characteristic of a measuring instrument that affects the result of a measurement performed by the instrument

3.18
influence quantity of a workpiece
characteristic of a workpiece that affects the result of a measurement performed on that workpiece

4 Symbols

For the purposes of this Technical Specification, the generic symbols given in Table 1 apply.

Table 1 — Generic symbols

Symbol	Description
a	limit value for a distribution
a_{xx}	limit value for an error or uncertainty contributor (in the unit of the result of measurement, of the measurand)
a^*_{xx}	limit value for an error or uncertainty contributor (in the unit of the influence quantity)
α	linear coefficient of thermal expansion
b	coefficient for transformation of a_{xx} to u_{xx}
C	correction (value)
d	resolution of a measurement equipment
E	Young's modulus
ER	error (value of a measurement)
G	function of several measurement values [$G(X_1, X_2, ... X_i, ...)$]
h	hysteresis value
k	coverage factor
m	number of standard deviations in the half of a confidence interval
MR	measurement result (value)
n	number of ...
N	number of iterations
ν	Poisson's number
p	number of total uncorrelated uncertainty contributors
r	number of total correlated uncertainty contributors
ρ	correlation coefficient
TV	true value of a measurement
u, u_i	standard uncertainty (standard deviation)
s_x	standard deviation of a sample
$s_{\bar{x}}$	standard deviation of a mean value of a sample
u_{c}	combined standard uncertainty
u_{xx}	standard deviation of uncertainty contributor xx — uncertainty component
U	expanded uncertainty of measurement
U_{A}	true uncertainty of measurement
U_{C}	conventional true uncertainty of measurement
U_{E}	approximated uncertainty of measurement (number of iteration not stated)
$U_{\mathrm{E}N}$	approximated uncertainty of measurement of iteration number N
U_{R}	required uncertainty
U_{T}	target uncertainty
U_{V}	uncertainty value (not estimated according to GUM or this Technical Specification)
X	measurement result (uncorrected)
X_i	measurement result (in the transparent box model of uncertainty estimation)
Y	measurement result (corrected)

5 Concept of the iterative GUM-method for estimation of uncertainty of measurement

Applying the GUM method completely one will find a conventional true uncertainty of measurement, U_C.

The simplified, iterative method/procedure of this Technical Specification is to achieve estimated uncertainties of measurements, U_E by overestimating the influencing uncertainty components/contributors ($U_E \geqslant U_C$). The process of overestimating provides "worst-case-contributions" at the upper bound from each known or predictable uncertainty contributor, thus ensuring results of estimations "on the safe side", i.e. not underestimating the uncertainty of measurement. The simplified, iterative method of this Technical Specification is based on the following:

— all uncertainty contributors are identified;

— it is decided which of the possible corrections shall be made (see 8.4.6);

— the influence on the uncertainty of the result of measurement from each contributor is evaluated as a standard uncertainty u_{xx}, called the uncertainty component;

NOTE As a convention in the iterative method the influence of each contributor must be converted into the unit of the measurand — using relevant physical equations/formulae and sensibility coefficients.

— an iteration process, PUMA (see clause 6);

— the evaluation of each of the uncertainty components (standard uncertainties) u_{xx} can take place either by type A-evaluation or by type B-evaluation;

— type B-evaluation is preferred — if possible — in the first iteration in order to get a rough uncertainty estimate to establish an overview and to save cost;

— the total effect of all contributors (called the combined standard uncertainty) is calculated by the formula:

$$u_c = \sqrt{u_{x1}^2 + u_{x2}^2 + u_{x3}^2 + \ldots + u_{xn}^2} \qquad (1)$$

— the formula (1) is only valid for a black box model of the uncertainty estimation and when the components u_{xx} are all uncorrelated (for more details and other formulas see 8.6 and 8.7);

— for simplification the only correlation coefficients between contributors considered are

$$\rho = 1, -1, 0 \qquad (2)$$

if the uncertainty components are not known to be uncorrelated, full correlation is assumed, either $\rho = 1$ or -1. Correlated components are added arithmetically before put into the formula above (see 8.5 and 8.6);

— the expanded uncertainty U is calculated by the formula:

$$U = k \times u_c \qquad (3)$$

where $k = 2$; k is the coverage factor (see also 8.8);

The simplified, iterative method normally will consist of at least two iterations of estimating the components of uncertainty.

a) The first very rough, quick and cheap iteration has the purpose of identifying the largest components of uncertainty (see Figure 1);

b) The following iterations — if any — only deal with making more accurate "upper bound" estimates of the largest components to lower the estimate of the uncertainty (u_c and U) to a possible acceptable magnitude.

The simplified and iterative method may be used for two purposes:

a) Management of the uncertainty of measurement for a result of a given measurement process (can be used for the results from a known measuring process or for comparison of two or more of such results) — see 6.2.

b) Uncertainty management for a measuring process. Development of an adequate measuring process i.e. $U_E \leqslant U_T$ — see 6.3.

6 Procedure for Uncertainty MAnagement — PUMA

6.1 General

The prerequisite for uncertainty budgeting and management is a clearly identified and defined measuring task; i.e. the measurand to be quantified (a GPS characteristic of a workpiece or a metrological characteristic of a GPS measuring equipment). The uncertainty of measurement is a measure of the quality of the measured value according to the definitions of a GPS characteristic of the workpiece or a metrological characteristic of the GPS measuring equipment given in GPS standards.

GPS standards define the "conventional true values" (see 1.20 of VIM:1993) of the characteristics to be measured by chains of standards and global standards (see ISO/TR 14638). GPS standards in many cases also define the ideal — or conventional true — principle of measurement (see 2.3 of VIM:1993), method of measurement (see 2.4 of VIM:1993), measurement procedure (see 2.5 of VIM:1003) and Standard "reference conditions" (see 5.7 of VIM:1993).

Deviations from the standardized conventional true values of the characteristics, etc. (the ideal operator) are contributing to the uncertainty of measurement.

6.2 Uncertainty management for a given measurement process

Management of the uncertainty of measurement for a given measuring task (box 1 of Figure 1) and for an existing measurement process is illustrated in Figure 1. The principle of measurement (box 3), measurement method (box 4), measurement procedure (box 5) and measurement conditions (box 6) are fixed and given or decided in this case, and cannot be changed. The only task is to evaluate the consequence on the uncertainty of measurement. A required U_R may be given or decided.

Using the iterative GUM method the first iteration is only for orientation, and to look for the dominant uncertainty contributors. The only thing to do — in the management process in this case — is to refine the estimation of the dominant contributors to come closer to a true estimate of the uncertainty components thus avoiding a too big overestimate — if necessary.

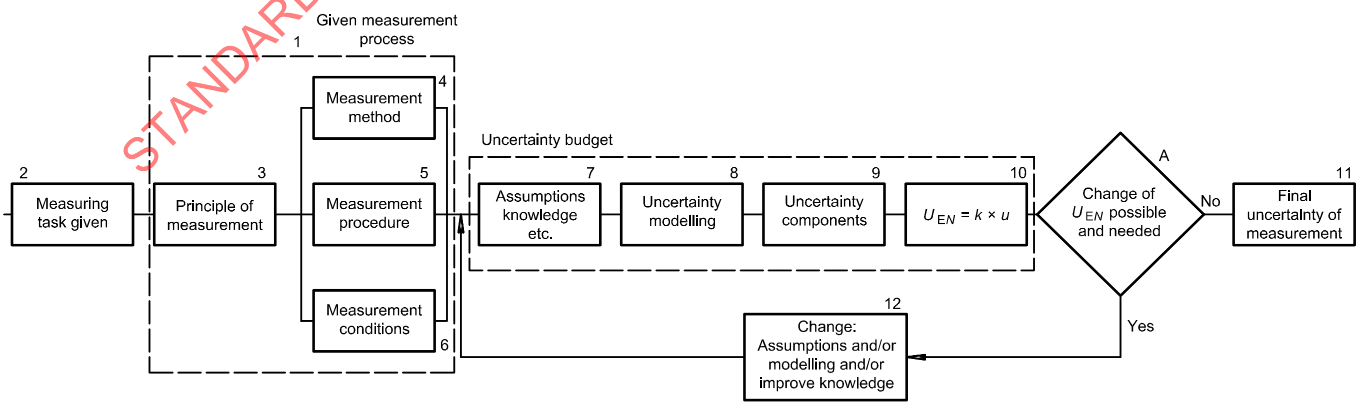


Figure 1 — Uncertainty management for a result of measurement from a given measurement process

The procedure is as follows:

a) make a first iteration based preferably on a black box model of the uncertainty estimation process and set up a preliminary uncertainty budget (boxes 7 to 9) leading to the first rough estimate of the expanded uncertainty, U_{E1} (box 10). For details about uncertainty estimation see 9. All estimates of uncertainties U_{EN} are performed as upper bound estimates;

b) compare the first estimated uncertainty, U_{E1}, with the required uncertainty U_R (box A) for the actual measuring task

   1) If U_{E1} is acceptable (i.e. if $U_{E1} \leqslant U_R$), then the uncertainty budget of the first iteration has proven that the given measurement procedure is adequate for the measuring task (box 11);

   2) If U_{E1} is not acceptable (i.e. if $U_{E1} > U_R$) or if there is no required uncertainty, but a lower and more true value is desired, the iteration process continues;

c) before the new iteration, analyze the relative magnitude of the uncertainty contributors. In many cases a few uncertainty components dominate the combined standard uncertainty and expanded uncertainty;

d) change the assumptions or improve the knowledge about the uncertainty components to make a more accurate (see 3.5 of VIM:1993) upper bound estimation of the largest (dominant) uncertainty components (box 12).

   Change to a more detailed model of the uncertainty estimation process or a higher resolution of the measuring process (box 12);

e) make the second iteration of the uncertainty budget (boxes 7 to 9) leading to the second, lower and more accurate (see 3.5 of VIM:1993) upper bound estimate of the uncertainty of measurement, U_{E2} (box 10);

f) compare the second estimated uncertainty U_{E2} (box A) with uncertainty required U_R for the actual measuring task

   1) if U_{E2} is acceptable (i.e. if $U_{E2} \leqslant U_R$), then the uncertainty budget of the second iteration has proven that the given measurement procedure is adequate to the measuring task (box 11);

   2) if U_{E2} is not acceptable (i.e. if $U_{E2} > U_R$), or if there is no required uncertainty, but a lower and more true value is desired, then a third (and possibly more) iteration(s) is (are) needed. Repeat the analysis of the uncertainty contributors [additional changes of assumptions, improve in knowledge, changes in modelling, etc. (box 12)] and concentrate on the currently largest uncertainty contributors;

g) when all possibilities have been used for making more accurate (lower) upper bound estimates of the measuring uncertainties without coming to an acceptable measuring uncertainty $U_{EN} \leqslant U_R$, then it is proven, that it is not possible to fulfil the given requirement U_R.

6.3 Uncertainty management for design and development of a measurement process/procedure

Uncertainty management in this case is performed to develop an adequate measurement procedure [measurement of the geometrical characteristics of a workpiece or the metrological characteristics of a measuring equipment (calibration)]. Uncertainty management is performed on the basis of a defined measuring task (box 1 in Figure 2) and a given target uncertainty, U_T (box 2 in Figure 2). Definition of the measuring task and target uncertainty are company policy decisions to be made at a sufficiently high management level. An adequate measurement procedure is a procedure which results in an estimated uncertainty of measurement less than or equal to the target uncertainty. If the estimated uncertainty of measurement is much less than the target uncertainty, the measurement procedure may not be (economically) optimal for performing the measuring task (i.e. the measurement process is too costly).

The PUMA, based on a given measuring task (box 1) and a given target uncertainty U_T (box 2), includes the following (see Figure 2):

a) choose the principle of measurement (box 3) on the basis of experience and possible measurement instruments present in the company;

b) set up and document a preliminary method of measurement (box 4), measurement procedure (box 5) and measurement conditions (box 6) on the basis of experience and known possibilities in the company;

c) make a first iteration based preferably on a black box model of the uncertainty estimation process and set up a preliminary uncertainty budget (boxes 7 to 9) leading to the first rough estimate of the expanded uncertainty, U_{E1} (box 10). For details about uncertainty estimation see clause 9. All estimates of uncertainties U_{EN} are performed as upper bound estimates;

d) compare the first estimated uncertainty, U_{E1}, with the given target uncertainty, U_T (box A);

   1) if U_{E1} is acceptable (i.e. if $U_{E1} \leqslant U_T$), then the uncertainty budget of the first iteration has proven that the measurement procedure is adequate for the measuring task (box 11);

   2) if $U_{E1} << U_T$, then the measurement procedure is technically acceptable, but a possibility may exist to change the method and/or the procedure (box 13) in order to make the measuring process more cost effective while increasing the uncertainty. A new iteration is then needed to estimate the resulting measurement uncertainty, U_{E2} (box 10);

   3) if U_{E1} is not acceptable (i.e. if $U_{E1} > U_T$), the iteration process continues, or it is concluded that no adequate measurement procedure is possible;

e) before the new iteration, analyze the relative magnitude of the uncertainty contributors. In many cases a few uncertainty components pre-dominate the combined standard uncertainty and expanded uncertainty;

f) if $U_{E1} > U_T$, then change the assumptions, the modelling or increase the knowledge about the uncertainty components (box 12) to make a more accurate (see 3.5 of VIM:1993) upper bound estimation of the largest (dominant) uncertainty components;

g) make the second iteration of the uncertainty budget (boxes 7 to 9) leading to the second, lower and more accurate (see 3.5 of VIM:1993) upper bound estimate of the uncertainty of measurement, U_{E2} (box 10);

h) compare the second estimated uncertainty U_{E2} with the given target uncertainty, U_T (box A);

   1) if U_{E2} is acceptable (i.e. if $U_{E2} \leqslant U_T$), then the uncertainty budget of the second iteration has proven that the measurement procedure is adequate for the measuring task (box 11);

   2) if U_{E2} is not acceptable (i.e. if $U_{E2} > U_T$) then a third (and possibly more) iteration(s) is (are) needed. Repeat the analysis of the uncertainty contributors (additional changes of assumptions, modelling and increase in knowledge (box 12)) and concentrate on the currently largest uncertainty contributors;

i) when all possibilities has been used for making more accurate (lower) upper bound estimates of the measuring uncertainties without coming to an acceptable measuring uncertainty $U_{EN} \leqslant U_T$, then a change of the measurement method or the measurement procedure or the conditions of measurement (box 13) is needed to (possibly) bring down the magnitude of the estimated uncertainty, U_{EN}. The iteration procedure starts again with a first iteration;

j) if changes in the measurement method or the measurement procedure or conditions (box 13) do not lead to an acceptable uncertainty of measurement, the final possibility is to change the principle of measurement (box 14) and start the above mentioned procedure again;

k) if change of the measuring principle and the related iterations described above do not lead to an acceptable uncertainty of measurement the ultimate possibility is to change the measuring task and/or target uncertainty (box 15) and start the above mentioned procedure again;

l) if change of measuring task or target uncertainty is not possible, it is demonstrated, that no adequate measurement procedure exists (box 16).

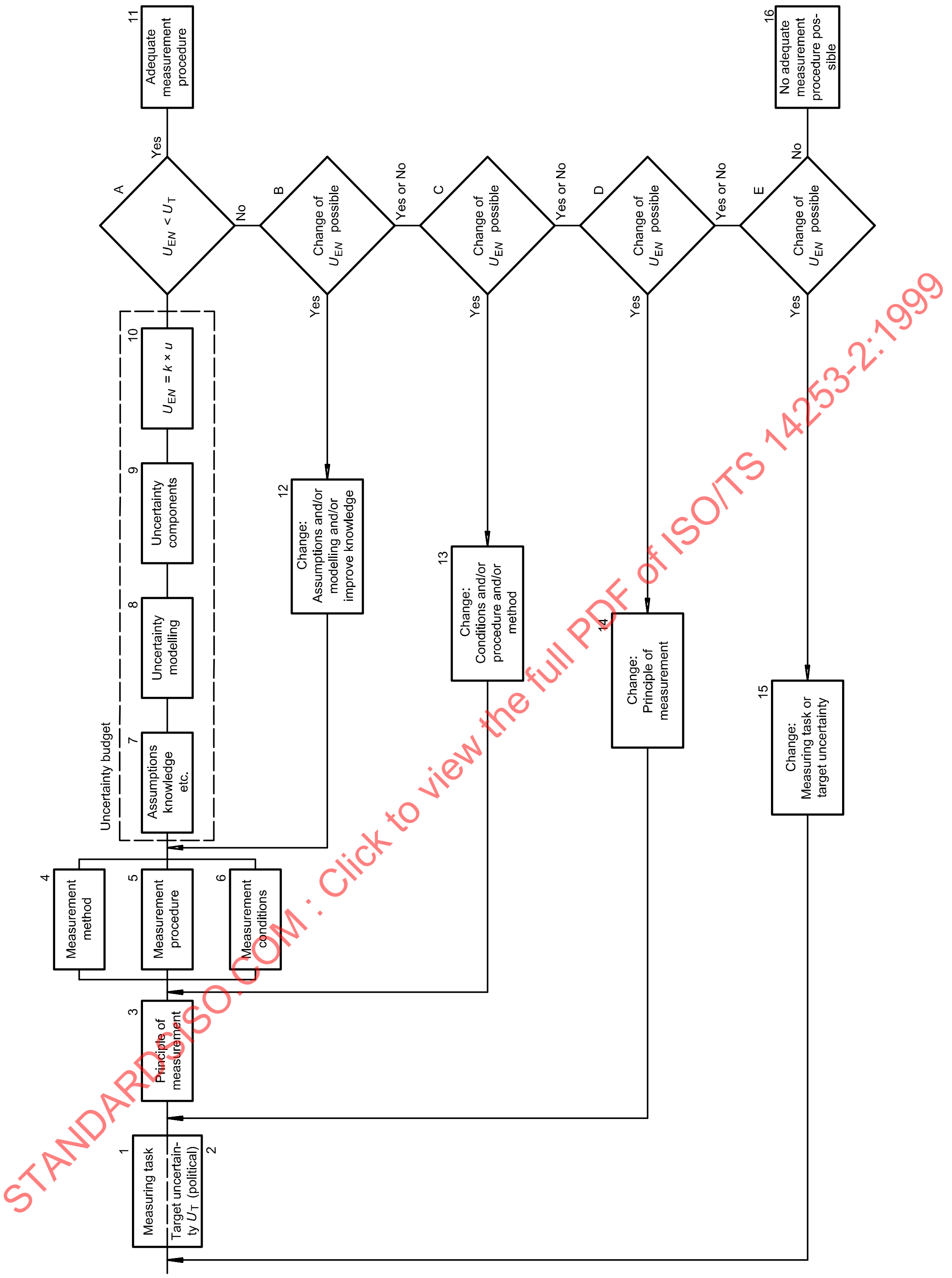


Figure 2 — Procedure for Uncertainty of Measurement MAnagement (PUMA) for a measurement process/procedure

7 Sources of errors and uncertainty of measurement

7.1 Types of errors

Different types of errors regularly shows up in measurement results.

— systematic errors;

— random errors;

— drift;

— outliers.

All errors are by nature systematic. When we see errors as non-systematic it is because the reason for the error is not looked for or because the level of resolution is not sufficient. Systematic errors may be characterised by size and sign (+ or –).

ER = MR – TV

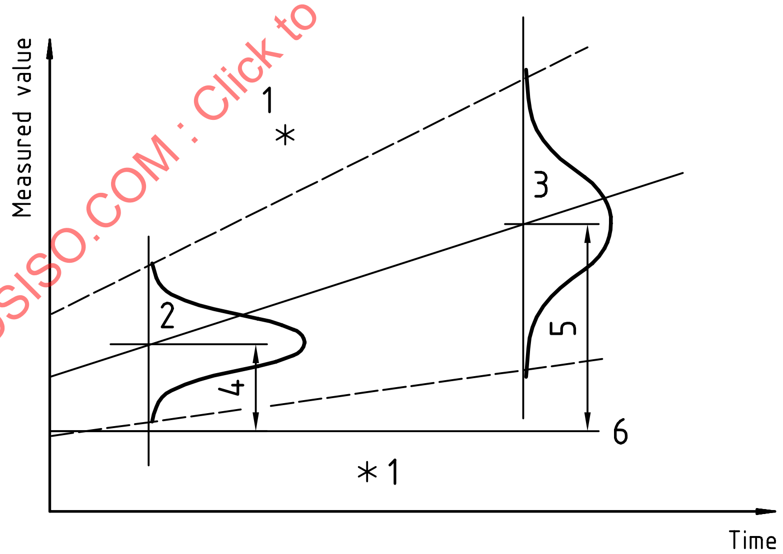
where

ER is the error,

MR is the measurement result;

TV is the true value.

Random errors are systematic errors caused by non-controlled random influence quantities. Random errors may be characterized by the standard deviation and the type of distribution. The mean value of the random errors is often considered as a basis for the evaluation of the systematic error (see Figure 3).



Key

1 Outlier
2 Dispersion 1
3 Dispersion 2
4 Systematic error 1
5 Systematic error 2
6 True value

Figure 3 — Types of errors in results of measurements

Drift is caused by a systematic influence of non-controlled influence quantities. Drift is often a time effect or a wear effect. Drift may be characterized by change per unit time or per amount of use.

Outliers are caused by not repeatable incidents in the measurement. Noise — electrical or mechanical — may result in outliers. A frequent reason for outliers is human mistakes as reading and writing errors or wrong handling of measuring equipment. Outliers are impossible to characterize in advance.

Errors or uncertainties in a measuring process will be a mix of known and unknown errors from a number of sources or error contributors.

The sources or contributors are not the same in each case, and the sum of the components are not the same.

It is still possible to make a systematic approach. There are always several sources or a combined effect of the ten different ones indicated in Figure 4.

In the following, examples and further details about each of the ten contributors are given.

What is often difficult is that each of the contributors may act individually on the result of measurement. But in many cases they even interfere with each other and cause additional errors and uncertainty.

Figure 4 and the following non-exhaustive lists (see 7.2 to 7.11) shall be used for getting ideas in a systematic way when making uncertainty budgets. In each case the evaluation of the actual error/uncertainty component needs knowledge about physics and/or experience in metrology.

In uncertainty budgets the uncertainty contributors and the uncertainty components may be grouped for convenience.

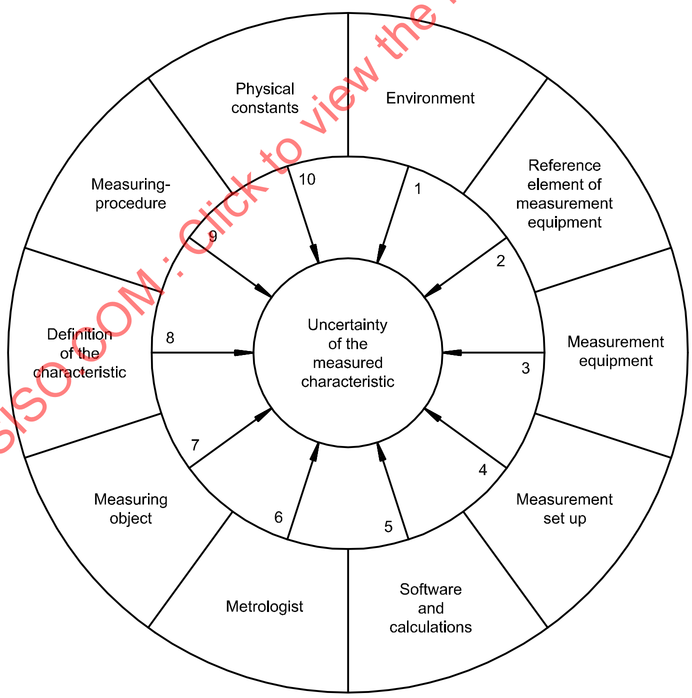


Figure 4 —Uncertainty contributors in measurement

7.2 Environment for the measurement

In most cases — especially in GPS measurements — the temperature is the main uncertainty contributor of the environment. Other uncertainty contributors may be:

- Temperature: absolute temperature, time variance, spatial gradient
- Vibration/noise
- Humidity
- Contamination
- Illumination
- Ambient pressure
- Air composition
- Air flow
- Gravity
- Electromagnetic interference
- Transients in the power supply
- Pressured air (e.g. air bearings)
- Heat radiation
- Workpiece
- Scale
- Instrument thermal equilibrium

7.3 Reference element of measurement equipment

The measuring equipment is divided into "reference element" and the "rest of the equipment", and it often pays to look at the equipment that way.

- Stability
- Scale mark quality
- Temperature expansion coefficient
- Physical principle: line scale, optical digital scale, magnetic digital scale, spindle, rack & pinion, interferometer
- CCD-techniques
- Uncertainty of the calibration
- Resolution of the main scale (analogue or digital)
- Time since last calibration
- Wavelength error

7.4 Measurement equipment

- Interpretation system
- Magnification, electrical or mechanical
- Error wavelength
- Zero-point stability
- Force stability/absolute force
- Hysteresis
- Guides/slideways
- Probe system
- Geometrical imperfections
- Stiffness/rigidity
- Reading system
- Linear coefficient for thermal expansion
- Temperature stability/sensitivity
- Parallaxes
- Time since last calibration
- Response characteristic
- Interpolation system, error wavelength
- Interpolation resolution
- Digitization

7.5 Measurement setup (excluding the placement and clamping of the workpiece)

In many cases there is no setup; the measurement equipment can measure "alone".

- Cosine errors and sine errors
- Abbe principle
- Temperature sensitivity
- Stiffness/rigidity
- Tip radius
- Form deviation of tip
- Stiffness of the probe system
- Optical aperture
- Interaction between workpiece and setup
- Warming up

7.6 Software and calculations

Observe that even the number of digits or decimals can have an influence!

- Rounding/Quantification
- Algorithms
- Implementation of algorithms
- Number of significant digits in the computation
- Sampling
- Filtering
- Correction of algorithm/Certification of algorithm
- Interpolation/extrapolation
- Outlier handling

7.7 Metrologist

The human being is not stable; there is a difference from day to day and often a rather large change during the day.

- Education
- Experience
- Training
- Physical disadvantages/ability
- Knowledge (precision, appreciation)
- Honesty
- Dedication

7.8 Measurement object, workpiece or measuring instrument characteristic

- Surface roughness
- Form deviations
- E-modulus (Young's modulus)
- Stiffness beyond E-modulus
- Temperature expansion coefficient
- Conductivity
- Weight
- Size
- Shape
- Magnetism
- Hygroscopic characteristic of the material
- Ageing
- Cleanliness
- Temperature
- Internal stress
- Creep characteristics
- Workpiece distortion due to clamping
- Orientation

7.9 Definition of the GPS characteristic, workpiece or measuring instrument characteristic

- Datum
- Reference system
- Degrees of freedom
- Toleranced feature
- ISO 4288
- Chain link 3 and 4 deviations (ISO/TR 14638)
- Distance
- Angle

7.10 Measuring procedure

- Conditioning
- Number of measurements
- Order of measurements
- Duration of measurements
- Choice of principle of measurement
- Alignment
- Choice of reference — reference item (standard) and value — relative to the measured value
- Choice of apparatus
- Choice of metrologist
- Number of operators
- Strategy
- Clamping
- Fixturing
- Number of points
- Probing principle and strategy
- Alignment of probing system
- Drift check
- Reversal measurements
- Multiple redundancy, error separation

7.11 Physical constants and conversion factors

- Knowledge of the correct physical values of, for example, material properties (workpiece, measuring instrument, ambient air, etc.)

8 Tools for the estimation of uncertainty components, standard uncertainty and expanded uncertainty

8.1 Estimation of uncertainty components

Estimation of uncertainty components can be done in two different ways. Type A evaluation and type B evaluation.

Type A-evaluation is evaluation of uncertainty components, u_{xx}, using statistical means. Type B evaluation is evaluation of uncertainty components, u_{xx}, by any other means than statistical.

Type A-evaluation will in most cases result in more accurate estimates of uncertainty components than type B-evaluation. In many cases Type B evaluation will result in sufficiently accurate estimations of uncertainty components.

Therefore, Type B evaluation shall be chosen in the iterative method, when it is not absolutely necessary to evaluate uncertainty by using type A evaluation. In a number of cases, no other possibilities exist than to use type A evaluation. See "standard cases" for evaluation of uncertainty components in 8.4.

NOTE The designation in this Technical Specification for both type A and B evaluated uncertainty components are u_{xx}. This is a deviation from the present version of GUM where type A evaluated uncertainty components is designated s_{xx} and type B-evaluated u_{xx}.

8.2 Type A evaluation for uncertainty components

8.2.1 General

Type A evaluation of the uncertainty component, u_{xx}, needs data from repeated measurements. The standard deviation of the distribution or the standard deviation of the mean value may be calculated using the formulas in 8.2.2.

8.2.2 Statistical tools

Regardless of the type of statistical distribution, the following statistical parameters are defined by the equations:

$\bar{x} = \frac{1}{n} \times \sum_{1}^{n} X_i$	The mean value of a number, n, of measurement results X_i. $\bar{x}$ is an estimate of the true value of the mean μ of the distribution.
$s_x = \sqrt{\frac{\sum_{1}^{n} (\bar{x} - X_i)^2}{(n-1)}}$	The standard deviation of the distribution of the sample based on n measurement values. s_x is an estimate of the standard deviation of the distribution σ.
$s_{\bar{x}} = \sqrt{\frac{\sum_{1}^{n} (\bar{x} - X_i)^2}{n \times (n-1)}} = \frac{s_x}{\sqrt{n}}$	The standard deviation of the mean value $s_{\bar{x}}$ of the sample is equal to the standard deviation of the sample divided by the square root of the number of measurements n.

When the mean value or the standard deviation is based on very few repeated measurements the estimated standard deviation values may be wrong, and possibly too small. For this reason, a "safety" factor h is used.

The safety factor h (calculated based on the Student t-factor)[2)] can be read from Table 1.

The standard deviation of the sample s_x (multiplied by the safety factor h as appropriate) is used in the uncertainty budget as the value for u_{xx}, when the measurement result is obtained using single readings of the component concerned.

$$u_{xx} = s_{x,n} \times h \tag{5}$$

The standard deviation of the mean value s is the value used for the standard uncertainty u_{xx} in the uncertainty budget when the measurement result is obtained using the mean of several readings of the component concerned.

$$u_{xx} = s_{\bar{x},n} \times h \qquad \left(s_{\bar{x},n} = \frac{s_{\bar{x},n} \times h}{\sqrt{n}} \right) \tag{6}$$

2) See also bibliographic reference [2].

Table 2 — Safety factors for standard deviations s_x of the sample

Number of measurements n	Safety factor h
2	7,0
3	2,3
4	1,7
5	1,4
6	1,3
7	1,3
8	1,2
9	1,2
$\leqslant 10$	1

8.3 Type B evaluation for uncertainty components

8.3.1 General

The evaluation of standard deviations by any means other than statistical is most often limited to previous experiences or by simply "guessing" what might be the standard deviation.

Experience shows that human beings do not "understand" or are not able to estimate standard deviations directly.

Experience shows that human beings remember limit values for variation (error limit values) or are able to develop such by using logical arguments and physical laws. In many cases specifications are known as limit values. This can be developed into a systematic method to derive standard deviations from limit values.

8.3.2 Transformation tools for error limits

Given a limit of variation, a. For all (limited) distributions there is a certain ratio between the standard deviation (defined by the same formula valid for all distributions, see 8.2.2) and the limit value, a. Then, if the limit value, a, is known and the type of distribution is known, it is possible to calculate the standard deviation. The limit value designation is chosen as $-a$ and $+a$ (only symmetrical distributions):

$$u_{xx} = a \times b \qquad (7)$$

Experience shows that in most cases it is sufficient to use only three types of distributions for transforming limits of variation into standard deviation.

In Figure 5 these three types of distribution are given with the formula for transforming from limit value to uncertainty component u_{xx} (standard uncertainty). The Gaussian distribution is not limited. Two times the standard deviation ($2s$) is used as the limit value for the Gaussian distribution. By experience it is known that a human being remembers the $2s$ value as the limit value for Gaussian distributed data. The b value for the three types of distribution in Figure 5 is:

— Gaussian: $b = 0,5$

— rectangular distribution: $b = 0,6$

— U-distribution: $b = 0,7$

Gauss. distribution: b = 0,5

$$u_{xx} = \frac{a}{2} \approx 0{,}5 \times a$$

Rectangular distribution: b = 0,6

$$u_{xx} = \frac{a}{\sqrt{3}} \approx 0{,}58 \times a \approx 0{,}6 \times a$$

U-distribution: b = 0,7

$$u_{xx} = \frac{a}{\sqrt{2}} \approx 0{,}71 \times a \approx 0{,}7 \times a$$

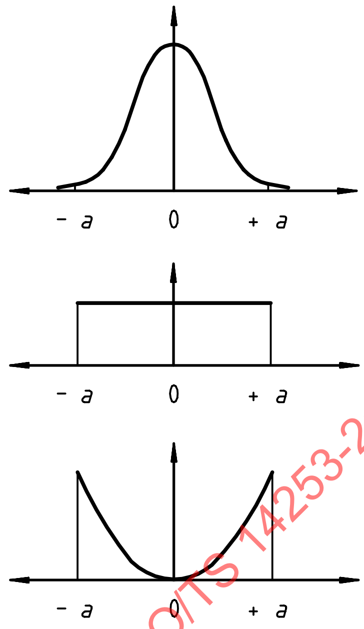


Figure 5 — The three types of distributions used for transforming limits of variation, a, into uncertainty components, u_{xx} (standard uncertainties)

Type B evaluation of the uncertainty component needs a reasonable "guess" or knowledge about the limit value, a. To be sure it is an overestimation make a high, but not too high guess of the limit value to determine the a value. Next step is to make an assumption about the distribution. In many cases the type of distribution is known or is obvious. If not, make a conservative assumption. If the distribution is not known to be Gaussian, then choose rectangular or U-distribution. If the type of distribution is not known to be rectangular, then choose U-distribution. The U-distribution is the most conservative assumption.

One way to make reasonable estimates of standard uncertainties — for influence quantities — without using statistical methods is by experience or by using physical laws to set up variation limits for a contributor and then transform these limit values to standard uncertainties by an assumed distribution type for the actual error/uncertainty component.

8.4 Common type A and B evaluation examples

8.4.1 General

In this clause some examples of common uncertainty contributors and components will be discussed. Examples will be given on how to derive the uncertainty component u_{xx}. The examples are not exhaustive for the problems arising in GPS measurement and calibration.

8.4.2 Experiment or limit value as basis for evaluation of the same uncertainty component

Data from repeated measurements give the possibility of using type A evaluation as well as type B evaluation of the resulting uncertainty component.

Data can be used to calculate the standard deviation (uncertainty component) using the formulas given in 8.2.2 (A-evaluation).

The same measured data may also be used in a B evaluation of the same uncertainty component only using the extreme values in the data-set as limit values (a values) around a mean. The uncertainty component is then calculated using the formulas in Figure 5.

8.4.3 Repeatability

In every uncertainty budget repeatability is involved at least one time. In most cases repeatability can only be evaluated by an experiment (type A evaluation). The uncertainty component is derived using the formulas for s_x and s given in 8.2.2.

The repeatability based uncertainty component may be less than the uncertainty component derived from the resolution of the measurement equipment reading. In this case the latter shall be used instead of the repeatability (see 8.4.4).

8.4.4 Resolution and rounding

The resolution of a measuring equipment (analogue or digital) or the step in last digit/decimal of a measured value or rounded measured value, whichever is the largest, is causing an uncertainty component:

$$u_{xx} = \frac{d}{2 \times \sqrt{3}} \approx \frac{d}{2} \times 0{,}6 \approx 0{,}3 \times d \tag{8}$$

where d is the resolution or the step in the last digit or decimal. The uncertainty component is equal to the component from a rectangular distribution with limit value $a = 0{,}5 \times d$.

When the repeatability uncertainty component is derived from experimental data, the effect from resolution, etc., is included if the repeatability uncertainty component is greater than the component based on resolution, etc.

8.4.5 Maximum permissible error (MPE) of a measuring equipment

When a measuring equipment or measuring standard is known to conform to stated MPE values for each of the metrological characteristics, these MPE values can be used to derive the related uncertainty components:

$$u_{xx} = \text{MPE} \times b \tag{9}$$

where b is chosen according to the rules given in 8.3.2 and the distribution assumed. When calibration data exist for one measuring equipment or for a larger number of identical pieces of equipment, it is often possible to use this data to find the type of distribution or even in rare cases to evaluate the uncertainty component directly — as an A-evaluation — by the formulas of 8.2.2.

8.4.6 Corrections

Errors, ER, where magnitude and sign (+ or –) is known may be compensated for by a correction, C, added to the measurement result:

$$C = -\text{ER} \tag{10}$$

Even when a correction is made, an uncertainty component (uncertainty of the correction) remains. This uncertainty component shall be less than the error/correction for the correction to have a positive effect on uncertainty of measurement.

It is the responsibility of the person who is making the uncertainty budget to decide if a known error shall be corrected for. The criteria to correct for a known error is based on economy.

Drift may be treated and dealt with as a known error, which may be corrected for.

8.4.7 Hysteresis

Hysteresis, h, in the indication of a measuring equipment may be treated as a symmetrical error/uncertainty around the mean of the two indications forming the hysteresis. The uncertainty component may be derived as an A-evaluation if sufficient data is present or as a B-evaluation where the uncertainty component is:

$$u_{xx} = \frac{h}{2} \times b \tag{11}$$

where b is chosen according to the rules given in 8.3.2 and the distribution assumed.

8.4.8 Influence quantities (temperature, measuring force, direction of measurement, etc.)

Measurements are influenced by a number of influencing quantities (see 2.7 of VIM:1993), which affects the measuring equipment and/or the object (e.g. component, measuring instrument etc.) being measured. Common influence quantities in GPS measurements are e.g. temperature, measuring force and direction of measurement. The influence is expressed in another physical unit than length [e.g. °C, N and ° (angle)] and shall be transformed by physical laws (equations) into length.

Influence quantities are often known as a value or a range and the uncertainty of the before mentioned value or range is known as a limit value.

8.4.8.1 Temperature

Standard reference temperature for GPS and GPS measurements is 20 °C (see ISO 1). Influences from temperature, which may be caused by absolute temperature as well as, time and spatial temperature gradients, result in linear expansion, bending, etc., of the measurement equipment, the measurement setup and the object being measured. The transformation from temperature to length is given by the linear expansion equation:

$$\Delta L = \Delta T \times \alpha \times L \tag{12}$$

where ΔT is the relevant temperature difference, α is temperature expansion coefficient of the material and L is the effective length under consideration.

In each case of temperature as an influence quantity several transformation equations from temperature to length may be in use together with other geometrical or physical equations to form the full description of the influence on the GPS measurement result (length, form, etc.).

8.4.8.2 Measuring force

Standard reference condition for GPS is zero measurement force. The effect on errors and uncertainty of length measurement by non-zero force is caused by elastic and in some cases also plastic deformation of the measurement equipment, the measurement setup and the measuring object. Especially the effect on the contact geometry between measuring equipment and measurement object shall be investigated.

The effect of measuring force may be quantified by experiments or by physical equations (Hertz formulas and others). The effect is depending on the force, the direction of the force, geometry and material constants such as E (Young's modulus), ν (Poisson's number), etc.

8.4.8.3 Direction of measurement

The direction of measurements shall be according to the definition of the geometrical characteristic of the measurement object (see ISO/TR 14638).

The effect of deviation from the defined directions of measurement can be calculated from basic trigonometric equations and be subject to the directional effects of the other influencing quantities.

8.4.9 Definition of the measurand

Measurands in GPS measurements are GPS characteristics of workpieces (often given as requirements on technical drawings) and metrological characteristics for measurement equipment and measurement standards.

These measurands are defined in GPS standards (see ISO/TR 14638 for an overview). In many cases the measurement procedure is intentionally or by accident not in conformance with the definition of the characteristic. In such cases these deviations in measurement procedure will result in errors and uncertainties in the result of measurement. If the errors are known, correction is possible (see 8.4.6). In practice a measurement procedure will always result in an uncertainty relative to the definition of the measurand (see also 8.4.11).

8.4.10 Calibration certificates

Calibration certificates give measured values for metrological characteristics and the related uncertainty of measurement. When the given calibrated value is used, the uncertainty component u_{xx} is derived as follows:

— the uncertainty is expressed as "expanded uncertainty", U, with a stated "coverage factor", k, according to GUM:

$$u_{xx} = \frac{U}{k} \tag{13}$$

Some calibration organizations have standardized a default value of k. In these cases, the "coverage factor" is not stated on the certificate;

— the uncertainty is expressed as a value U_{V} and a stated "confidence level", e.g. 95 % or 99 %:

$$u_{xx} = \frac{U_{\mathrm{V}}}{m} \tag{14}$$

where m is the number of standard deviations in the confidence interval corresponding to the stated confidence level.

Calibration certificates sometimes only — or in addition — certify, that the equipment fulfil a defined specification (a set of MPEs) given e.g. in a standard, manufacturers data sheet, etc. In this case the nominal MPE value of the metrological characteristic shall be used and the uncertainty component derived from this MPE value given in the specification according to 8.4.5.

8.4.11 Surface texture, form and other geometrical deviations of a measurement object

The surfaces of a measuring object is in contact with the measuring equipment during measurement. Depending on the surface texture, form deviations and other geometrical deviations from nominal geometry, the contact geometry (stylus tip) of the measuring equipment will interact with the surface and cause uncertainty components.

These components may be evaluated by experiments (A-evaluation) or B-evaluation or partly by experiments and partly by B-evaluation.

8.4.12 Physical constants

Physical constants (e.g. temperature expansion coefficients, Young's modulus, Poisson's number, etc.) which is part of corrections for or transformation from the influence quantity error or evaluated uncertainties are often not known accurately, but are estimated.

They are therefore introducing additional uncertainty components using the same transformation formulas as used for influence quantities above. This evaluation can only be done as B-evaluation.

8.5 Black and transparent box model of uncertainty estimation

The uncertainty for the same measurement process can in many cases be evaluated on several levels of detail or models. The two extreme cases are the black box and transparent box method.

In the black box method the total measurement process is modelled as a black box with unknown content. The uncertainty budget and the uncertainty components are only describing the total effect on the measurement process. In this choice of model it may be very difficult to determine the functional relationship between uncertainty components and individual error contributors.

To have the full benefit of uncertainty budgeting it may be necessary to open the black box and make a more detailed uncertainty budget. This could either be based on several smaller black boxes or the behaviour of all the details in the measurement process, the transparent box model of uncertainty estimation. The black box may also be characterized as a low resolution method and the transparent box method as a high resolution method/model.

In the black box model for uncertainty estimation, the input and output units are the same and the uncertainty components are assumed to be additive and the sum of the uncertainty components have the expectation value zero. For the purpose of the black box model in this technical report and the PUMA method, all influence quantities are transformed to the unit of the measurand. Therefore, in the black box model the sensitivity coefficients of the individual uncertainty component are equal to 1 (one).

In the transparent box model for uncertainty estimation these restrictions of the uncertainty components (additive uncertainty components, input unit the same as output unit and sensitivity coefficient equal to 1) are not valid.

8.6 Black box method of uncertainty estimation — Summing of uncertainty components into combined standard uncertainty, u_c

In the black box method of uncertainty estimation the result of the measurement is the reading corrected by an eventually known correction:

$$Y = X + C \tag{15}$$

where X is the reading of the measuring instrument and $C = \Sigma C_i$ is the sum of the corresponding additive corrections known from e.g. calibration, temperature correction, deformation correction, etc.

The combined standard uncertainty of measurement is given by the equation:

$$u_c = \sqrt{u_r^2 + \sum_1^p u_i^2} \tag{16}$$

where

p is the number of uncorrelated uncertainty contributors;

u_r is "the sum" of the strongly correlated ($\rho = 1$ and -1) uncertainty contributors, calculated by the equation:

$$u_r = \sum_1^r u_i \tag{17}$$

where r is the number of strongly correlated uncertainty contributors.

In total there are $p + r$ uncertainty contributors in measurement of Y.

The uncorrelated ($\rho = 0$) uncertainty contributors are to be added geometrically (the square root of the sum of squares).

The strongly correlated uncertainty contributors are to be added arithmetically.

A conservative estimate is to consider all uncertainty contributors which are known not to be fully uncorrelated as strongly correlated.

8.7 Transparent box method of uncertainty estimation — Summing of uncertainty components into combined standard uncertainty, u_c

In the transparent box method of uncertainty estimation the value of the measurand is modelled as a function of several measured values X_i, which themselves could be functions (transparent box models) and/or black box models:

$$Y = G\left(X_1, X_2, \ldots, X_i, \ldots, X_{p+r}\right) \quad (18)$$

The combined standard uncertainty of measurement is given by the equation:

$$u_c = \sqrt{u_r^2 + \sum_{i=1}^{p}\left(\frac{\partial Y}{\partial X_1} \times u_{Xi}\right)^2} \quad (19)$$

where u_r is the "sum" of the strongly correlated components of measuring uncertainty:

$$u_r = \sum_{i=1}^{r} \frac{\partial Y}{\partial X_i} \times u_{Xi} \quad (20)$$

where

$\frac{\partial Y}{\partial X_i}$ is the partial differential coefficient of the function Y with respect to X_i.

u_{Xi} is the combined standard uncertainty of measurement of the number i measured value (function), which is part of the transparent box method of uncertainty estimation for the measurement of Y.

u_{Xi} may be the result (u_c — combined standard uncertainty) of either a black box (see 8.6) or another transparent box method of uncertainty estimation.

The uncorrelated (ρ = 0) components of measuring uncertainty shall be added geometrically (the square root of the sum of squares).

The strongly correlated components of uncertainty shall be added arithmetically (the number of strongly correlated components of uncertainty is r).

A conservative estimate is to take as strongly correlated all components which are not known to be fully uncorrelated.

The number of uncorrelated components of uncertainty is p.

In total there have been $p + r$ components of uncertainty in this transparent box method of uncertainty estimation of Y, which again — each of them — could be a combination of a number of components of uncertainty of measurement.

8.8 Evaluation of expanded uncertainty, U, from combined standard uncertainty, u_c

The expanded uncertainty of measurement, U, in GPS measurements is calculated as:

$$U = u_c \times k = u_c \times 2 \quad (21)$$

Unless otherwise specified, the coverage factor k = 2 in GPS measurements (see ISO 14253-1).

8.9 Nature of the uncertainty of measurement parameters u_c and U

The uncertainty components and the combined uncertainty of measurement are, as shown, estimated as a standard uncertainty u_{xx} and u_c respectively. In practical industrial GPS measurements, the uncertainty components are a mix of constant and varying contributors with time constants covering several orders of magnitude. The uncertainty of measurement includes all systematic errors, which are not corrected for, regardless of the reason. It is impossible to correct for all systematic errors.

Therefore, in most cases, u_c and U are not stochastic variables. They represent quasi-constant, but not known errors. U and u_c shall, therefore, not be treated as standard deviations, but as constant (unknown) errors.

9 Practical estimation of uncertainty — Uncertainty budgeting with PUMA

9.1 General

The use of the PUMA method and how to make uncertainty budgets and related documentation are given as examples in annex A.

This clause only gives the sequence in the documentation and procedure of estimating **each of the components of uncertainty** to be put in an uncertainty budget.

9.2 Preconditions for an uncertainty budget

Setting up an uncertainty budget is only possible when:

— the measuring task is properly defined. The characteristic of the feature of the workpiece or the characteristic of the measurement equipment shall be defined and pointed out as the task (box 1 in Figure 2).

  An uncertainty budget is set up for one single specified measuring result only. One single measuring result may be taken as the representative for a group of measurement results;

— the measurement principle is properly defined and known, or at least known initially as a draft (box 3 in Figure 2);

— the measurement method is properly defined and known, or at least known initially as a draft (box 4 in Figure 2);

— the measurement procedure is properly documented and known, or at least known initially as a draft (box 5 in Figure 2).

  The measurement procedure include the choice of measurement equipment.

  The measurement procedure gives all the details of how the measuring equipment and the workpiece is handled during measurement. The uncertainty budget is mirroring the activities and steps in the procedure;

— the measurement conditions are defined and known, or at least known initially as a draft (box 6 in Figure 2).

Observe that every measurement will include the three elements (1, 2 and 3) illustrated in Figure 6. The uncertainty budget shall reflect the three elements:

— determination of a reference point (1 in Figure 6), often a zero point. In many cases the zero point 7of the measurement equipment is set as an activity in the calibration procedure. Uncertainty is related to the setting of the reference point or zero point;

— determination of a measuring point (2 in Figure 6), the reading of the measurement equipment when measuring the characteristic of the workpiece or measurement equipment. Uncertainty is related to the reading itself depending on characteristics of the equipment and the measuring object;

— a travel of the measurement equipment (3 in Figure 6) from the reference point to the measurement point. The error and/or uncertainty of this travel is known from the calibration of the equipment.

Each of the three elements is again and additionally influenced by the error sources given in clause 7. The influence from the error/uncertainty sources shall be systematically checked in the uncertainty budget.

Key

1 Reference point
2 Measuring point
3 Travel of measuring equipment
a Uncertainty range of reference point
b Uncertainty range of measuring point

Figure 6 — Generic model of the three elements in a measurement

The overall measuring task, i.e. the characteristic to be quantified (measured) is often evaluated as a simple calculation based on two or more measured values, or basic measurements of the same kind, i.e. when the error of indication of an equipment is characterized by the error range. In such cases the uncertainty budget may be set up for the basic measurement, e.g. one of several calibration values. The uncertainty related to the characteristic to be quantified is evaluated by calculations based on the uncertainty value of the basic measurement.

9.3 Standard procedure for uncertainty budgeting

The following procedure may be helpful for setting up and documenting of an uncertainty budget, first iteration of the PUMA method:

9.3.1 Define and document the overall measuring task (characteristic to be measured) and the basic measurement value [basic measurement result (see 9.2)] for which the uncertainty budget shall be set up.

9.3.2 Document:

— measurement principle,

— measurement method,

— measurement procedure,

— measurement conditions.

If not fully known, choose and document initial or assumed draft principle, draft method, draft procedure and draft conditions in accordance with the principle of overestimation of uncertainty components given in clause 5.

9.3.3 Make a graphical presentation of the measurement setup(s). The figure(s) may be of help for understanding the uncertainty contributors present in the measurement.

9.3.4 Document the mathematical relations between measured values and the characteristics of the overall measuring task.

The mathematical relation is normally not needed when the measuring task can be solved by a black box method (see 8.6).

The mathematical relation is needed when the measuring task shall be solved by a transparent box method (see 8.7).

9.3.5 Make an initial investigation and documentation of all possible uncertainty contributors and components. The result and the documentation may be stated in a table as illustrated in Figure 7.

The investigation is made in a systematic sequence using the three elements given in Figure 6, the potential error sources given in clause 7 and the already documented information of 9.3.1 and 9.3.2.

The subdivision of the uncertainty of measurement into uncertainty components should be done in a way that does not include the same component more than once, but in many practical cases this is not possible. The principle is most important for the dominant components in an uncertainty budget.

Designation (low resolution)	Designation (high resolution)	Name	Comments (initial)
u_{xx}	$u_{x\mathrm{a}}$	Name of *xa*	Initial observations, information, comments and decisions related to uncertainty component *xa*
	$u_{x\mathrm{b}}$	Name of *xb*	Initial observations, information, comments and decisions related to uncertainty component *xb*
	$u_{x\mathrm{c}}$	Name of *xc*	Initial observations, information, comments and decisions related to uncertainty component *xc*
		Name of total *xx*	Initial observations, information, comments and decisions related to uncertainty component total *xx*
u_{yy}	$u_{y\mathrm{a}}$	Name of *ya*	Initial observations, information, comments and decisions related to uncertainty component *ya*
	$u_{y\mathrm{b}}$	Name of *yb*	Initial observations, information, comments and decisions related to uncertainty component *yb*
		Name of total *yy*	Initial observations, information, comments and decisions related to uncertainty component total *yy*
u_{zz}		Name of *zz*	Initial observations, information, comments and decisions related to uncertainty component *zz*

Figure 7 — Initial overview, designation, naming and commenting on the uncertainty components of an uncertainty budget

The table in Figure 7 has two levels of resolution. These levels are useful in the initial phase and before the first PUMA iteration, where the modelling of the uncertainty is no yet established. Low resolution often means one single black box as the model. High resolution gives the possibility of splitting the single black box into several smaller black boxes.

For each uncertainty component define and document mathematical designations and names (labels) on the two levels of resolution.

Use the comments column in Figure 7 to sum up information, conditions and even initial decisions related to the actual uncertainty component. The comments column is a note pad!

9.3.6 Based on the information present and documented in Figure 7 investigate and establish for the uncertainty modelling for the actual iteration step.

For each uncertainty component:

— decide on the evaluation method, type A or B evaluation (see 8.2 and 8.3);

— document and argue for the evaluation of the uncertainty component value, the background, etc.;

— in case of type A evaluation, state the component value and the number of measurements on which it is based;

— in case of type B evaluation, state the limit value a^* (variation limit in the unit of the influence quantity), a, the assumed distribution and the resulting uncertainty component value.

9.3.7 Investigate, search for and document any possible correlation between the documented uncertainty components in accordance with clause 5.

9.3.8 Choose the correct formulas depending on modelling and correlation and calculate the combined standard deviation, u_c (see 8.6 and 8.7).

9.3.9 Derive the expanded uncertainty, U, where $U = 2 \times u_c$ (see 8.8).

9.3.10 Make a summary table containing all key information in the uncertainty budget (see example in Figure 8). Investigate possible changes which may change the uncertainty estimate — to be ready for the next iteration — if necessary now or later. Especially make an economical evaluation.

Component name	**Evaluation type**	**Distribution type**	**Number of measurements**	**Variation limit** a^* [influence units]	**Variation limit** a [µm]	**Correlation coefficient**	**Distribution factor** b	**Uncertainty comp.** u_{xx} [µm]
u_{Xa} Name of *xa*	A		10			0		1,60
u_{Xb} Name of *xb*	B	Gaussian		1,90	1,90	0	0,5	0,95
u_{Xc} Name of *xc*	B	Rectangular		3,42	3,42	0	0,6	2,05
u_{Ya} Name of *ya*	A		15			0		1,20
u_{Yb} Name of *yb*	A		15			0		0,60
u_{Za} Name of *za*	B	U		10 °C	157	0	0,7	1,10
u_{Zb} Name of *zb*	B	U		15 °C α_1/α_2= 1,1	60	0	0,7	0,42
Combined standard uncertainty, u_c								**3,29**
Expanded uncertainty (k = 2), U								**6,58**

Figure 8 —Example of a summary table with all key information of an uncertainty budget

10 Applications

10.1 General

A normal uncertainty budgeting for a GPS measurement may result in the following equation. The uncertainty components are grouped depending on their origin:

$$u_c = \sqrt{u_{MPEx}^2 + \ldots + u_{Mx}^2 + \ldots + u_{Bx}^2 + \ldots + u_{Ox}^2 + \ldots u_{Ex}^2 + \ldots} \tag{22}$$

$$U = u_c \times k \; (k = 2) \tag{23}$$

The groups of uncertainty components originate from, for example:

— measurement equipment (or measurement standard)	u_{MPE1}, u_{MPE2}, u_{MPE3}, ...
— environment	u_{M1}, u_{M2}, u_{M3}, ...
— personnel/staff	u_{B1}, u_{B2}, u_{B3}, ...
— measurement set up	u_{O1}, u_{O2}, u_{O3}, ...
— measurement object (workpiece or measurement equipment)	u_{E1}, u_{E2}, u_{E3}, ...
— definition of the characteristic of the object	u_{D1}, u_{D2}, u_{D3}, ...
— measurement procedure	u_{P1}, u_{P2}, u_{P3}, ...
— etc.	$u_{etc.x}$, ...

Experience is that the different groups of uncertainty components in many cases are not influencing each other when the changes in one of the other groups are small. This means, that the equation can be used to evaluate the influence from one or more of the groups on the uncertainty of measurement, absolute as well as relative.

It is possible also to "transform" the uncertainty budget and the changes in one or more of the groups into economical terms and effect, and thus use the uncertainty budget to evaluate the economical influence of the uncertainty components.

In the following sub-clauses, applications of uncertainty budgets and the PUMA method are given. The list is non-exhaustive.

10.2 Documentation and evaluation of the uncertainty value

As demonstrated in many cases through this Technical Specification, the uncertainty budget is able to give an estimate of the uncertainty value for an existing measurement or calibration process.

10.3 Design and documentation of the measurement or calibration procedure

10.3.1 Documentation and optimization of measurement and calibration processes

The PUMA method gives the opportunity of documenting and optimizing a measurement or a calibration process by taking into account technical and/or economical criteria, when optimizing through a number of iterations.

10.3.2 Development of measurement procedures and instructions

Development of measurement procedures and uncertainty budgets in parallel, the PUMA method gives the opportunity of analyzing the effect of every sub-procedure based on the effect on the uncertainty. Thus develop (and optimize) the total measurement procedure and the related instruction.

10.3.3 Development of calibration procedures and instructions

Development of calibration procedures and uncertainty budgets in parallel, the PUMA method gives the opportunity of analyzing the effect of every sub-procedure based on the effect on the uncertainty. Thus develop (and optimize) the total calibration procedure and the related instruction.

10.3.4 Qualification or disqualification of secondary measurement methods and equipment

In many cases the ideal measuring method and measurement equipment — according to the definition of the characteristic to be measured (GPS characteristic of a workpiece or metrological characteristic of a measurement equipment) — is too expensive and/or slow. Results of analysis of the measuring object for form and angular deviations and investigation of the influence on the uncertainty budget gives the possibility of qualifying or disqualifying secondary measurement methods and equipment and cut costs, e.g. investigate if a three point measurement (secondary method) in a V-block may be a valid substitute for measurement of roundness by variation in roundness (ideal method in accordance with the definition of roundness).

10.3.5 Qualification of measurement equipment and set ups

The influence on the uncertainty of measurement from a specific measurement equipment ($u_{\text{MPE}x}$) and measurement set up ($u_{\text{O}x}$) can be seen from the uncertainty budget. All other uncertainty components are taken as invariable. When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the equipment and the set up are qualified for the measurement task.

10.3.6 Demonstration of best measuring capability, BMC

The Best Measuring Capability (BMC) is the least possible uncertainty of measurement achievable in a company or a laboratory for a specific measuring task. When all uncertainty components in an uncertainty budget are minimized, $u_{\text{c,min}}$ is the BMC for the task.

10.4 Design, optimization and documentation of the calibration hierarchy

10.4.1 Design of the calibration hierarchy

The uncertainty budget results in an equation which gives a functional relation between two levels in the calibration hierarchy in a company or in a calibration laboratory (see example in annex A and Figure 9). Use of the PUMA method — with a stated "target uncertainty" — on representatively shop floor measurements with the uncertainty components originating from the measurement equipment ($u_{\text{MPE}x}$) as variables — and all other uncertainty components as fixed values — results in minimum requirements (MPEs) for the metrological characteristics of the measurement equipment (see Figure 9).

The same procedure used on the calibration measurements of the measurement equipment will result in minimum requirements for the metrological characteristics of the measurements standards. The procedure can be used at all levels of the calibration hierarchy and thus design the full hierarchy in a company or a laboratory.

10.4.2 Requirements for and qualification of measurement standards

The influence on the uncertainty of measurement in calibration from a specific measurement standard ($u_{\text{MPE}x}$) can be seen from the uncertainty budget. All other uncertainty components are taken as invariable. When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the measurement standard is qualified for the calibration task.

10.4.3 Requirements for and qualification of external calibration certificates

The metrological characteristics of the reference standards in a company or laboratory result in uncertainty components in the uncertainty budgets for calibration of the next lower level of the calibration hierarchy. The reference standards are acting as "measurement equipment", the equipment at the next lower level is acting as measurement object. Taking all other uncertainty components as invariable and the uncertainty components from the reference standard ($u_{\mathrm{MPE}x}$) as variables the requirements to the calibration certificates can be derived from the formula:

$$u_{\mathrm{T}} \geqslant u_{\mathrm{c}} = \sqrt{u_{\mathrm{EMT}x}^2 + \ldots + u_{\mathrm{M}x}^2 + \ldots u_{\mathrm{B}x}^2 + \ldots + u_{\mathrm{O}x}^2 + \ldots u_{\mathrm{E}x}^2 + \ldots + u_{\mathrm{D}x}^2 + \ldots u_{\mathrm{P}x}^2 + \ldots +} \quad (24)$$

When the resulting combined standard uncertainty fulfils the target uncertainty requirement the calibration certificate is qualified.

10.4.4 Evaluation of the use of check standards

Check standards used in the workshop — as an addition to calibration — may be a way to decrease the uncertainty of measurement. By substitution of the relevant uncertainty components in the original uncertainty budget, based on the calibrated measurement equipment, and adding possible new uncertainty components, the effect of a check standard on the uncertainty of measurement can be evaluated (see the example in annex A).

10.5 Design and documentation of new measurement equipment

10.5.1 Specification for a new measurement equipment

The uncertainty budget for a specific measuring task can be set up with the uncertainty components from the measurement equipment ($u_{\mathrm{MPE}x}$) as unknown variables and all other uncertainty components as invariable. The requirements for a new measurement equipment, which does not exist yet in the company, can be derived from formula (24).

10.5.2 Design of special measurement equipment

The uncertainty budget for a specific measuring task can be set up with the uncertainty components from the not yet designed measurement equipment as unknown variables and all other uncertainty components as invariable. The design requirements for the new measurement equipment can be derived from formula (24).

10.6 Requirements for and qualification of the environment

The influence on the uncertainty of measurement from the environment ($u_{\mathrm{M}x}$) can be seen from the uncertainty budget. All other uncertainty components are invariable. The uncertainty components from the environment are taken as variables. It is then possible to derive requirements for the environment from formula (24).

When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the environment is qualified for the measurement task.

10.7 Requirements for and qualification of measurement personnel

The influence on the uncertainty of measurement from the personnel ($u_{\mathrm{B}x}$) can be seen from the uncertainty budget. All other uncertainty components are invariable. The uncertainty components from the personnel are taken as variables. It is then possible to derive requirements for the personnel from formula (24).

When the resulting combined standard uncertainty fulfils the target uncertainty requirement, the personnel is qualified for the measurement task.

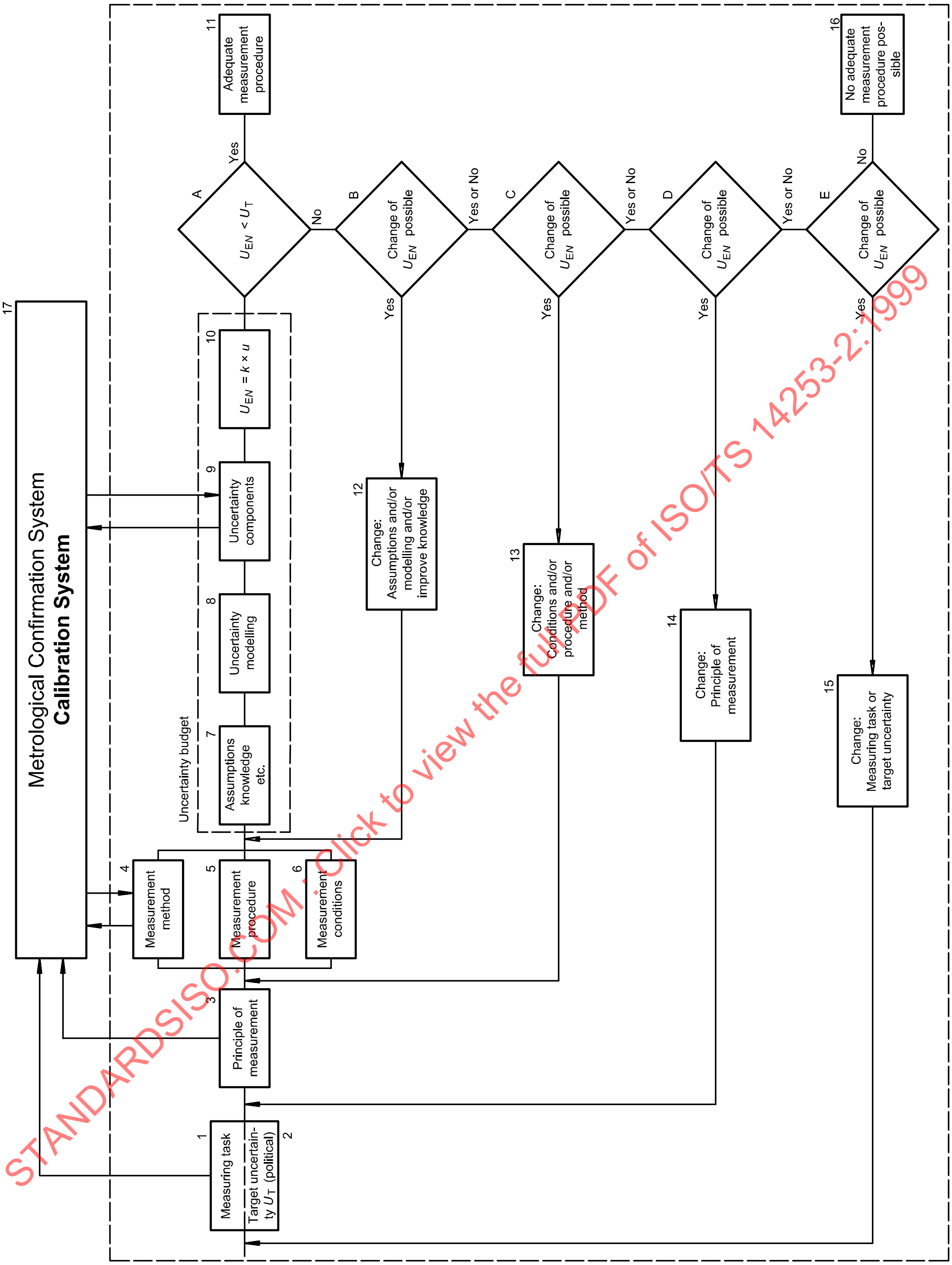


Figure 9 — Relationship between the uncertainty budget and the calibration level for the measurement equipment or measurement standard used in the measurement

Annex A
(informative)

Example of uncertainty budgets — Calibration of a setting ring

WARNING — It shall be recognized that the following example is constructed to illustrate the PUMA only. It only includes uncertainty contributors significant in the illustrated cases. For different target uncertainties and applications, other uncertainty contributors may be significant.

A.1 Scope

This example covers the estimation of uncertainty of measurement and qualification of a measurement procedure and measurement conditions for a measurement task using the PUMA method.

A.2 Task and target uncertainty

A.2.1 Measuring task

The measuring task consists of calibrating a ∅ 100 mm × 15 mm setting ring, two point diameter in one defined direction in the symmetry plane. The roundness in the symmetry plane is 0,2 µm.

A.2.2 Target uncertainty

The target uncertainty is 1,5 µm.

A.3 Principle, method, procedure and condition

A.3.1 Measurement principle

Mechanical contact, comparison with a known length (reference ring).

A.3.2 Measurement method

Differential, comparison of a ∅ 100 mm reference standard and the "unknown" ∅ 100 mm setting ring.

A.3.3 Initial measurement procedure

— The setting ring is measured on a horizontal measuring machine.

— A reference ring (∅ 100 mm) is used.

— The horizontal measuring machine is used as a comparator.

A.3.4 Initial measurement conditions

— Horizontal measuring machine is within manufacturers specification (see Table A.1).

— Digital step in the read out display: 0,1 µm.

— Temperature in the laboratory is 20 °C ± 1 °C.

— The temperature variation of the measuring machine over time is registered to 0,25 °C.

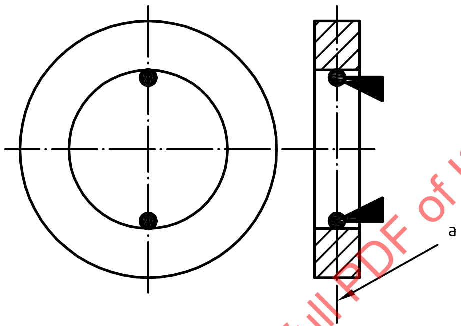
— The temperature difference between setting ring and reference ring is less than 1 °C.

— The measuring machine and the rings are made of steel.

— The operator is trained and familiar with the use of the measuring machine.

A.4 Graphical illustration of measurement setup

See Figure A.1.



a Symmetry plane

Figure A.1 — Measurement setup

A.5 List and discussion of the uncertainty contributors

See Table A.1.

Table A.1 — Overview and comments table for uncertainty components in diameter measurements

<table>
<tr><th>Designation
Low res.</th><th>Designation
High res.</th><th>Name
Uncertainty component</th><th colspan="2">Comments</th></tr>
<tr><td>u_{RS}</td><td></td><td>Reference standard (ring)</td><td colspan="2">The uncertainty is stated for the ∅ 100 mm diameter on the (accredited) calibration certificate as U = 0,8 µm.</td></tr>
<tr><td>u_{EC}</td><td></td><td>Error of indication of the measuring machine</td><td colspan="2">The measuring machine is calibrated and is documented within the specifications (MPE values). The scale error is within: 0,6 m + 4,5 µm/m for a floating zero.</td></tr>
<tr><td>u_{PA}</td><td></td><td>Alignment of measuring anvils</td><td colspan="2">Since the reference ring and the ring to be calibrated are contacted the same way (as long as their diameters are within a reasonable range), the parallelism error can be neglected.</td></tr>
<tr><td>u_{RR}</td><td>u_{RA}</td><td>Resolution</td><td>$u_{RA} = \frac{d}{2 \times \sqrt{3}} = \frac{0{,}1\ \mu m}{2 \times \sqrt{3}} \approx 0{,}029\ \mu m$</td><td rowspan="2">The largest of the two = u_{RR}</td></tr>
<tr><td></td><td>u_{RE}</td><td>Repeatability</td><td>A repeatability study has been conducted. The standard deviation is found to be 0,7 µm. (this corresponds to 0.5 µm for measuring the master ring and 0,5 µm for measuring the gage ring, when squared together).</td></tr>
<tr><td>u_{TD}</td><td></td><td>Temperature difference between the two rings</td><td colspan="2">The temperature difference between the master ring and the ring being calibrated is assumed to follow a U-shaped distribution. It is assumed that the two measurements are so close together in time that the measuring machine does not change temperature.</td></tr>
<tr><td>u_{TA}</td><td></td><td>Difference in temperature expansion coefficients</td><td colspan="2">The temperature is assumed to follow a U-shaped distribution. It is assumed that the two measurements are so close together in time that the measuring machine does not change temperature.</td></tr>
<tr><td>u_{RO}</td><td></td><td>Roundness of setting ring</td><td colspan="2">The roundness is measured as 0,2 µm. The ring has an elliptical shape error.</td></tr>
</table>

A.6 First iteration

A.6.1 First iteration — Documentation and calculation of the uncertainty components

u_{RS} — **Reference standard (ring)** **Given in calibration certificate**

According to the calibration certificate (Certificate no. XPQ-23315-97) the expanded uncertainty of the certified diameter of the reference ring is 0,8 µm (coverage factor k = 2):

$$u_{RS} = \frac{U}{k} = \frac{0{,}8\ \mu m}{2} = 0{,}8\ \mu m \times 5 = 0{,}4\ \mu m$$

u_{EC} — **Error of indication of the horizontal measuring machine** **Type B evaluation**

The MPE value of the error of indication curve (based on floating zero) is 0,6 µm + 4,5 µm/m. The measurement distance (difference in diameter) between the reference ring and the ring calibrated is very small (<< 1 mm). Therefore:

$$a_{EC} = 0{,}6\ \mu m$$

For safety reasons, a rectangular distribution (b = 0,6) is assumed. This results in an uncertainty component of:

$$u_{EC} = 0{,}6\ \mu m \times 0{,}6 = 0{,}36\ \mu m$$

u_{PA} — **Alignment of measuring anvils** **Type B evaluation**

Since the reference ring and the setting ring to be calibrated are contacted the same way (as long as their diameters are within a reasonable range), the parallelism error can be neglected.

$$u_{PA} \approx 0\ \mu m$$

u_{RR} **— Repeatability/resolution** **Type A evaluation**

A repeatability study has been conducted on the difference of ring diameters. The standard deviation is found to be 0,7 µm. (This corresponds to 0,5 µm for measuring the master ring and 0,5 µm for measuring the gauge ring, when squared together.)

This gives an uncertainty component of:

$$s_{RR} = \frac{0{,}7\ \mu m}{6} = 0{,}12\ \mu m$$

u_{TD} **— Temperature difference between the two rings** **Type B evaluation**

The temperature difference between the two rings is not seen to be greater than 1 °C. The temperature expansion coefficient for the two rings is assumed equal α = 1,1 µm/(100 mm × °C). This means:

$$a_{TD} = 1{,}1 \frac{\mu m}{(100\ mm \times\ ^{\circ}C)} \times 1\ ^{\circ}C \times 100\ mm = 1{,}1\ \mu m$$

A U-distribution is assumed (b = 0,7):

$$u_{TD} = 0{,}11\ \mu m \times 0{,}7 = 0{,}77\ \mu m$$

u_{TA} **— Difference in temperature expansion coefficients** **Type B evaluation**

The deviation from 20 °C is maximum 1 °C. The difference in temperature expansion coefficients is assumed to be less than 10 %. Therefore:

$$a_{TD} = 1{,}1 \frac{\mu m}{(100\ mm \times\ ^{\circ}C)} \times 1\ ^{\circ}C \times 100\ mm \times 10\ \% = 0{,}11\ \mu m$$

A U-distribution is assumed (b = 0,7):

$$u_{TA} = 0{,}11\ \mu m \times 0{,}7 \approx 0{,}08\ \mu m$$

u_{RO} **— Roundness of the setting ring** **Type B evaluation**

The form error is elliptical and the out of roundness is 0,2 µm. The diameter is defined and measured in one specified direction in the ring. Therefore the roundness has no significant effect.

$$u_{RO} \approx 0\ \mu m$$

A.6.2 First iteration — Correlation between uncertainty components

It is estimated that no correlation occurs between the uncertainty components.

A.6.3 First iteration — Combined and expanded uncertainty

When no correlation between the uncertainty components, the combined standard uncertainty is:

$$u_c = \sqrt{u_{RS}^2 + u_{EC}^2 + u_{PA}^2 + u_{RR}^2 + u_{TD}^2 + u_{TD}^2 + u_{RO}^2}$$

The values from A.1.6.1:

$$u_c = \sqrt{\left(0{,}40^2 + 0{,}36^2 + 0^2 + 0{,}12^2 + 0{,}77^2 + 0{,}08^2 + 0^2\right)\ \mu\text{m}^2}$$

$$u_c = 0{,}95\ \mu\text{m}$$

Expanded uncertainty:

$$U = u_c \times k = 0{,}95\ \mu\text{m} \times k = 1{,}90\ \mu\text{m}$$

A.6.4 Summary of uncertainty budget — First iteration

See Table A.2.

Table A.2 — Summary of uncertainty budget (first iteration)

Component name	Evaluation type	Distribution type	Number of measurements	Variation limit a^* [influence units]	Variation limit a [µm]	Correlation coefficient	Distribution factor b	Uncertainty comp. u_{xx} [µm]
u_{RS} Reference standard (ring)	Cert.					0	0,5	0,40
u_{EC} Error of indication of the measuring machine	B	Rect.		0,6 µm	0,6	0	0,6	0,36
u_{PA} Alignment of measuring anvils	B	Rect.		0 µm	0	0	0,6	0
u_{RR} Repeatability/resolution	A		6			0		0,12
u_{TD} Temperature difference between the two rings	B	U		1 °C	1,1	0	0,7	0,77
u_{TA} Difference in temperature expansion coefficients	B	U		1 °C	0,11	0	0,7	0,08
u_{RO} Roundness of setting ring	B			0 µm	0	0		0
Combined standard uncertainty, u_c								**0,95**
Expanded uncertainty (k = 2), U								**1,90**

A.6.5 First iteration — Discussion of the uncertainty budget

The criterion $U_{E1} < U_T$ is not met. There is one dominant uncertainty component, u_{TD}, caused by the temperature difference of 1 °C. It is not possible to make a smaller estimate u_{TD} by the existing information. The only solution is to change the measurement conditions. The temperature acclimatization shall be better, that means more time for the acclimatization and probably a more efficient heat protection from body parts of the operator during handling and measurement.

Change (decrease) of other uncertainty components — other than the temperature related uncertainty components — in the uncertainty budget will have nearly no effect on the combined standard deviation and the expanded uncertainty.

A.6.6 Conclusion on the first iteration

The measurement procedure is qualified by the first iteration, but the measurement conditions need improvement.

The maximum temperature difference between the two rings shall not exceed 0,5 °C.

A.7 Second iteration

The temperature conditions are changed from 1 °C to 0,5 °C in the formulas for u_{TD} and u_{TA} (see A.6.1). Documentation and calculation of the uncertainty components shall be changed accordingly.

Table A.3 — Summary of uncertainty budget (second iteration)

Component name	Evaluation type	Distri-bution type	Number of measure-ments	Variation limit a^* [influence units]	Variation limit a [µm]	Correlation coefficient	Distri-bution factor b	Uncer-tainty comp. u_{xx} [µm]
u_{RS} Reference standard (ring)	Cert.					0	0,5	0,40
u_{EC} Error of indication of the measuring machine	B	Rect.		0,6 µm	0,6	0	0,6	0,36
u_{PA} Alignment of measuring anvils	B	Rect.		0 µm	0	0	0,6	0
u_{RR} Repeatability/resolution	A		6			0		0,12
u_{TD} Temperature difference between the two rings	B	U		0,5 °C	0,55	0	0,7	0,39
u_{TA} Difference in temperature expansion coefficients	B	U		0,5 °C	0,06	0	0,7	0,04
u_{RO} Roundness of setting ring	B			0 µm	0	0		0
Combined standard uncertainty, u_c								**0,67**
Expanded uncertainty (k = 2), U								**1,35**
NOTE The change in uncertainty components is indicated by thick lines.								

A.8 Conclusion on the second iteration

In the second iteration, the temperature difference is limited to 0,5 °C. Table A.3 gives the documentation, the target uncertainty criterion is met:

$$u_{E2} = 1{,}35\ \mu m \leqslant U_T = 1{,}5\ \mu m$$

By the second iteration, the measurement conditions are qualified.

A.9 Comments — Summary of example

By the example it is demonstrated that it is possible to qualify a measurement procedure and a set of measurement conditions using the PUMA method to fulfil a given target uncertainty criterion:

$$U_{EN} \leqslant U_T$$

After the first iteration, where the target uncertainty criterion is not met, it is — in this case — obvious what to do. There is only one dominant uncertainty component. The temperature conditions shall be better to meet the target uncertainty criterion. It is demonstrated how the individual uncertainty contributor influence the combined standard uncertainty and expanded uncertainty after the first iteration. Depending on the relative size of the uncertainty components a strategy for a decreasing of the uncertainty can be made.

Annex B
(informative)

Example of uncertainty budgets — Design of a calibration hierarchy

WARNING — It shall be recognized that the following example is constructed to illustrate the PUMA only. It only includes uncertainty contributors significant in the illustrated cases. For different target uncertainties and applications, other uncertainty contributors may be significant.

B.1 Scope

This example illustrates how the PUMA method may be used in industry to optimize and plan in detail the metrological (calibration) hierarchy. The example include:

— measurement of local diameter with external micrometer;

— calibration of an external micrometer;

— calibration requirements for measurement standards for calibration of an external micrometer;

— use of check standard as a supplement to calibration.

Furthermore, it includes the estimation of uncertainty of measurement and evaluation of the requirements for metrological characteristics at the lover three levels of the traceability hierarchy shown in Figure B.1. These three levels are:

III Measurement of the local (two-point) diameter of a cylinder using an external micrometer. The measurement procedure is evaluated by the PUMA method and a given target uncertainty U_T (see clause B.2).

II Calibration of the metrological characteristics (which influence the uncertainty of measurement in sub-example I) of an external micrometer (see clauses B.3, B.4 and B.5).

I Calibration requirements (MPE values) for the metrological characteristics of the calibration standards needed for calibration of the external micrometer (see clause B.6).

Use of a check standard as a supplement to calibration of the external micrometer is evaluated by the uncertainty budget as a variant of the measurement of two point diameter (see clause B.7).

At level III, the uncertainty of measurement for the two-point diameter measurement is evaluated. The maximum permissible errors (MPEs) of the metrological characteristics of the external micrometer [MPE_{ML} (error of indication), MPE_{MF} (flatness of measuring anvils), and MPE_{MP} (parallelism of measuring anvils)] are taken as unknown variables. From the function:

$$U_T \geqslant U_{WP} = f(MPE_{ML}, MPE_{MF}, MPE_{MP}, \text{other uncertainty contributors})$$

the MPE values for the three metrological characteristics (MPE_{ML}, MPE_{MF}, and MPE_{MP}) of the external micrometer can be derived. At level II, the uncertainty of measurement in calibration of the three metrological characteristics (error of indication, flatness of measuring anvils and parallelism of measuring anvils) is estimated. At level I, the MPE values for the metrological characteristics of the three measurement standards are derived with the same technique used for the MPEs of the micrometer, but now taking the MPE values of the three measurement standards as unknown variables.

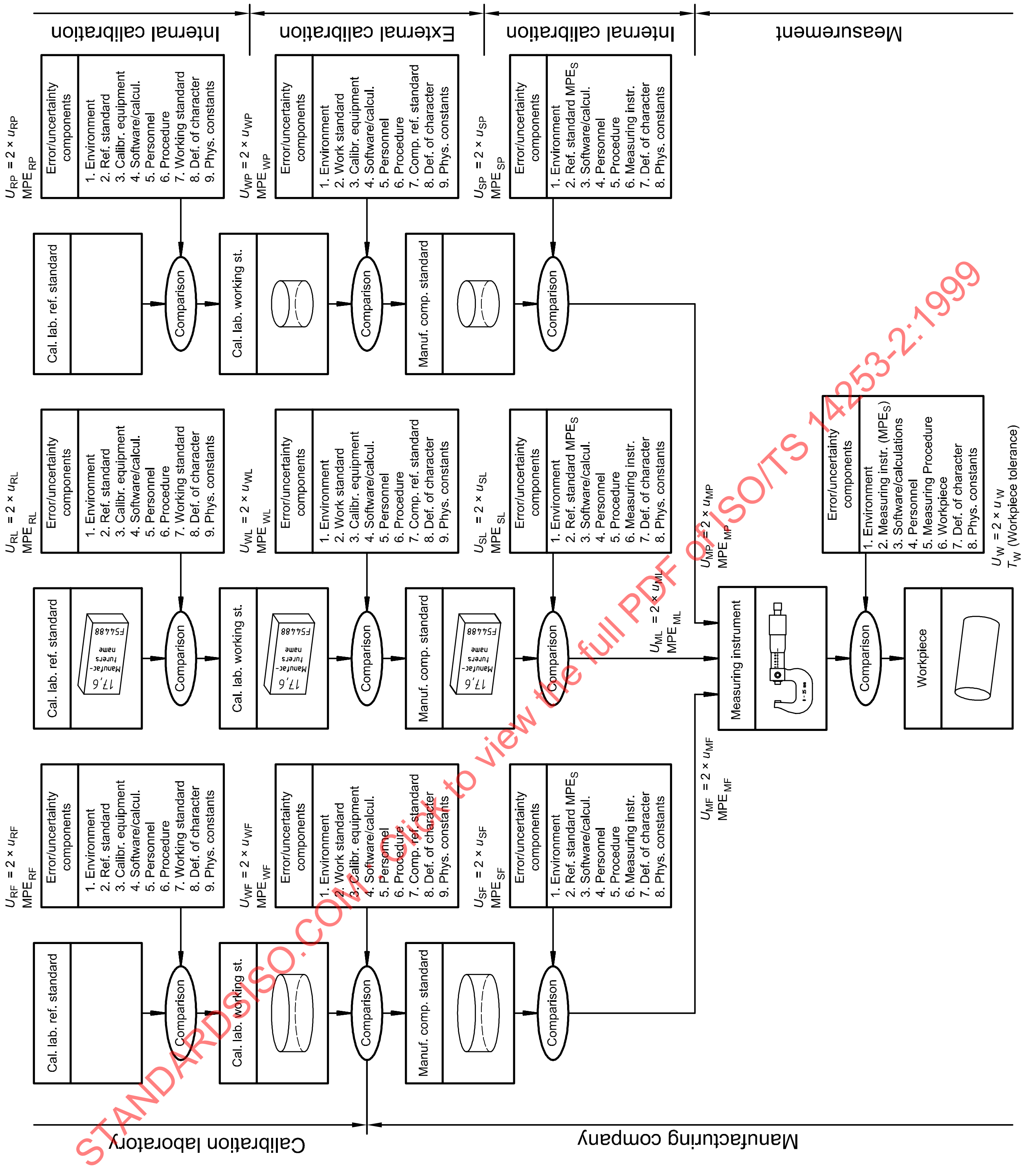


Figure B.1 — Calibration hierarchy for measurement of local diameter and calibration of external micrometers

The result of uncertainty budgeting on the three levels is:

— the MPE values for the external micrometer are optimized and directly derived from the need for uncertainty of measurement on the workshop floor;

— the MPE values for the measurement standards (gauge blocks, optical flat and optical parallels) are optimized to calibration of the above external micrometer. These MPE values are the minimum requirements to calibration certificates;

— the improvement of the uncertainty of measurement using a check standard as a supplement to calibration can be quantified.

B.2 Measurement of local diameter

B.2.1 Task and target uncertainty

B.2.1.1 Measuring task

The measuring task consists of measuring the local diameter (two-point diameter) on a series of fine turned steel shafts, with nominal dimensions ∅ 25 mm × 150 mm.

B.2.1.2 Target uncertainty

The target uncertainty is 8 µm.

B.2.2 Principle, method and conditions

B.2.2.1 Measurement principle

Measurement of length — Comparison with a known length.

B.2.2.2 Measurement method

The measurements are performed with an analogue external micrometer with flat (∅ 6 mm) measuring anvils, measuring range 0 to 25 mm with a vernier scale interval of 1 µm.

B.2.2.3 Initial measurement procedure

— The diameter is measured while the shaft is still clamped in the chuck of the machine tool.

— Only one measurement of the diameter is allowed.

— The shaft is cleaned with a cloth before measurement.

— The friction/ratchet drive shall be used during measurements.

— The spindle clamp shall not be used.

B.2.2.4 Initial measurement conditions

— It is demonstrated that the temperature in the shafts and in the micrometer is varying during time. The maximum deviation from standard reference temperature 20 °C is 15 °C.

— Maximum temperature difference between the shafts and the micrometer is 10 °C.

— Three different operators are using the machine tool and the micrometer for the production of the shafts.

— The cylindricity of the shafts is found to be better than 1,5 µm.

— The type of form error is not known, except that the conicity is small.

B.2.3 Graphical illustration of the measurement setup

See Figure B.2.

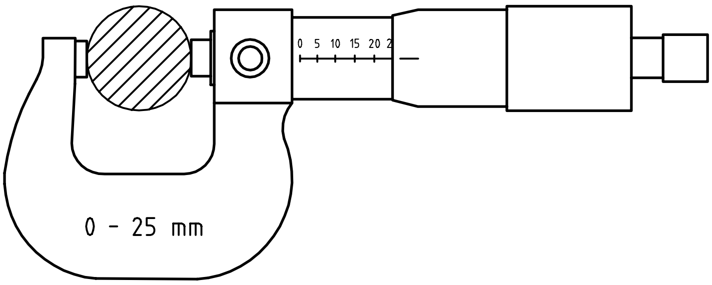


Figure B.2 — Measurement setup for measurement of local ∅ 25 mm diameter

B.2.4 List and discussion of the uncertainty contributors

The two-point diameter measurement is modelled as a black box uncertainty estimation process. No corrections are used. All error contributions are included in the uncertainty of measurement.

In Table B.1 all the uncertainty contributors are mentioned and named, which is assumed to influence the uncertainty of the actual diameter measurements.

Table B.1 — Overview and comments table for uncertainty components in measurement of local diameter (two-point diameter)

Designation Low resolution	Designation High resolution	Name Uncertainty component	Comments	
u_{ML}		Micrometer — Error of indication	Requirement for error of indication MPE_{ML} of the micrometer is an unknown variable. Initially it is set to 6 µm — and symmetrical positioning of the error of indication curve by zero adjustment after calibration.	
u_{MF}		Micrometer — Flatness of measuring anvils	Requirement for out of flatness for the two measuring anvils M_{MF} is an unknown variable. Initially it is set to 1 µm.	
u_{MP}		Micrometer — Parallelism of measuring anvils	Requirement for out of parallelism between the two measuring anvils M_{MP} is an unknown variable. Initially it is set to 2 µm.	
u_{MX}		Effect of spindle clamping, orientation of the micrometer and time of handling	These effects are in this case not active. The spindle clamp is not used. The orientation and time of handling have no significant effect on a 0 to 25 mm micrometer.	
u_{RR}	u_{RA}	Resolution	$u_{RA} = \frac{d}{2 \times \sqrt{3}} = \frac{1\,\mu m}{2 \times \sqrt{3}} = 0{,}29\ \mu m$	The largest of the two = u_{RR}
	u_{RE}	Repeatability	It is demonstrated by experiments, that the three operators have the same repeatability. The experiment includes more than 15 measurements for each operator on "perfect" Ø 25 mm plug gauges. The effect of the flexibility of the micrometer is included in the repeatability.	
u_{NP}		Variation of zero point between three operators	The three operators are using the micrometer in a different way. The zero point is not the same as set by the calibration "person". Experiment (more than 15 measurements for each operator on "perfect" Ø 25 mm plug gauges).	
u_{TD}		Temperature difference	Maximum difference, between shafts and micrometer, seen during observation period is 10 °C.	
u_{TA}		Temperature	Maximum deviation from standard reference temperature (20 °C) is 15 °C.	
u_{WE}		Workpiece form error	Cylindricity measured is 1,5 µm. The major part of the cylindricity is out of roundness. The effect on diameter is two times the cylindricity, 3 µm.	

B.2.5 First iteration

B.2.5.1 First iteration — Documentation and calculation of the uncertainty components

u_{ML} — Micrometer — Error of indication **Type B evaluation**

MPE_{ML} for the metrological characteristic error of indication of an external micrometer is usually defined as the maximum range of the error of indication curve, and not related to the zero error of indication. Position of the error of indication curve to zero error is another (independent) metrological characteristic.

In this case it is assumed that the error of indication curve is positioned — during the calibration procedure — so that the largest negative and positive error of indication is of the same absolute value.

The definitive value of MPE_{ML} is not fixed yet. It is one of the tasks of the uncertainty budget. As an initial setting of MPE_{ML}, 6 µm is chosen. Because of the zero setting procedure mentioned the error limit value is:

$$a_{ML} = \frac{6\ \mu m}{2} = 3\ \mu m$$

A rectangular distribution is assumed (overestimation principle, because Gaussian distribution cannot be proved on the given basis) (b = 0,6):

$$u_{ML} = 3\ \mu m \times 0{,}6 = 1{,}8\ \mu m$$

u_{MF} — Micrometer — flatness of measuring anvils **Type B evaluation**

The flatness deviation is active in diameter measurements on shafts, while the calibration of the error of indication curve is performed on gauge blocks with plane and parallel surfaces.

The definitive value of MPE_{MF} is not fixed yet. It is one of the tasks of the uncertainty budget. As an initial setting of MPE_{MF}, 1 µm is chosen.

MPE_{MF} is influencing the uncertainty budget twice, once for each of the two measuring anvils. A Gaussian distribution is assumed (b = 0,5):

$$u_{MF} = 1\ \mu m \times 0{,}5 = 0{,}5\ \mu m$$

u_{MP} — Micrometer — parallelism of measuring anvils **Type B evaluation**

The parallelism deviation is active in diameter measurements on shafts, while the calibration of the error of indication curve is performed on gauge blocks with plane and parallel surfaces.

The definitive value of MPE_{MP} is not fixed yet. It is one of the tasks of the uncertainty budget. As an initial setting of MPE_{MF}, 2 µm is chosen. A Gaussian distribution is assumed (b = 0,5):

$$a_{MP} = 2\ \mu m$$

$$u_{MP} = 2\ \mu m \times 0{,}5 = 1\ \mu m$$

u_{RR} — Repeatability/Resolution **Type A evaluation**

All three operators have the same repeatability. It is tested in an experiment, where ∅ 25 mm plug gauges have been used as "workpieces". Hence the form error from the real workpieces is not included in the repeatability study. All operators have performed 15 measurements. The common standard deviation is

$$u_{RR} = 1{,}2\ \mu m$$

The resolution uncertainty component, u_{RA}, is included in u_{RR}, in this case ($u_{RA} < u_{RE}$).

u_{NP} — Variation of zero point between three operators **Type A evaluation**

From the same experiments used for repeatability the differences in zero-point between the three operators and the calibration personnel are investigated:

$$u_{NP} = 1\ \mu m$$

u_{TD} — Temperature difference **Type B evaluation**

The temperature difference between micrometer and workpieces is observed to maximum 10 °C. There is no information about which of them has the highest temperature. Therefore ± 10 °C is assumed. The linear coefficient of thermal expansion, α, is assumed to be 1,1 µm/(100 mm × °C) for the micrometer and the workpieces. The limit value is:

$$a_{TD} = \Delta T \times \alpha \times D = 10\,^\circ\mathrm{C} \times 1{,}1 \frac{\mu\mathrm{m}}{(100\ \mathrm{mm} \times {}^\circ\mathrm{C})} \times 25\ \mathrm{mm} = 2{,}8\ \mu\mathrm{m}$$

A U-distribution is assumed (b = 0,7):

$$u_{TD} = 2{,}8\ \mu\mathrm{m} \times 0{,}7 = 1{,}96\ \mu\mathrm{m}$$

u_{TA} — Temperature **Type B evaluation**

The observed maximum deviation from standard reference temperature (20 °C) is 15 °C. There is no information about the sign of this deviation, therefore ± 15 °C is assumed. A 10 % maximum difference between the two linear coefficients of thermal expansion ($\alpha_{micrometer}$ and $\alpha_{workpiece}$) is assumed. The limit value is:

$$a_{TA} = 0{,}1 \times \Delta T_{20} \times \alpha \times D = 0{,}1 \times 15\,^\circ\mathrm{C} \times 1{,}1 \frac{\mu\mathrm{m}}{(100\ \mathrm{mm} \times {}^\circ\mathrm{C})} \times 25\ \mathrm{mm} = 0{,}4\ \mu\mathrm{m}$$

A U-distribution is assumed (b = 0,7):

$$u_{TA} = 0{,}4\ \mu\mathrm{m} \times 0{,}7 = 0{,}28\ \mu\mathrm{m}$$

u_{WE} — Workpiece form error **Type B evaluation**

The cylindricity is measured on a sample of shafts and found to be 1,5 µm. Cylindricity is a measure for the variation of radius. The effect on the diameter is assumed to be two times the cylindricity deviation, while no information exists to make it smaller. The limit value is:

$$a_{WE} = 3\ \mu\mathrm{m}$$

A rectangular distribution is assumed (b = 0,6):

$$u_{WE} = 1{,}8\ \mu\mathrm{m}$$

B.2.5.2 First iteration — Correlation between uncertainty components

It is estimated: no correlation between the uncertainty components.

B.2.5.3 First iteration — Combined and expanded uncertainty

When no correlation between the uncertainty components, the combined standard uncertainty is:

$$u_c = \sqrt{u_{ML}{}^2 + u_{MF}{}^2 + u_{MF}{}^2 + u_{MP}{}^2 + u_{RR}{}^2 + u_{NP}{}^2 + u_{TD}{}^2 + u_{TA}{}^2 + u_{WE}{}^2}$$

The values from B.2.5.1:

$$u_c = \sqrt{\left(1{,}8^2 + 0{,}5^2 + 0{,}5^2 + 1{,}0^2 + 1{,}2^2 + 1{,}0^2 + 1{,}96^2 + 0{,}28^2 + 1{,}8^2\right)\ \mu\mathrm{m}^2}$$

$$u_c = 3{,}79\ \mu\mathrm{m}$$

$$U = u_c \times k = 3{,}79\ \mu\mathrm{m} \times 2 = 7{,}58\ \mu\mathrm{m}$$

B.2.5.4 Summary of uncertainty budget — First iteration

See Table B.2

Table B.2 — Summary of uncertainty budget (first iteration) — Measurement of two-point diameter

Component name		Evaluation type	Distri-bution type	Number of measure-ments	Variation limit a^* [influence units]	Variation limit a [µm]	Correlation coefficient	Distribution factor b	Uncer-tainty comp. u_{xx} [µm]
u_{ML}	Micrometer — error indication	B	Rect.		3,0 µm	3,0	0	0,6	1,80(1)
u_{MF}	Micrometer — flatness 1	B	Gaussian		1,0 µm	1,0	0	0,5	0,50(3)
u_{MF}	Micrometer — flatness 2	B	Gaussian		1,00 µm	1,0	0	0,5	0,50(3)
u_{MP}	Micrometer — parallelism	B	Gaussian		2,0 µm	2,0	0	0,5	1,00(2)
u_{RR}	Repeatability	A		15			0		1,20(2)
u_{NP}	Variation of 0-point	A		15			0		1,00(2)
u_{TD}	Temperature difference	B	U		10 °C	2,8	0	0,7	1,96(1)
u_{TA}	Temperature	B	U		15 °C $\alpha_1/\alpha_2 = 1{,}1$	0,4	0	0,7	0,28(3)
u_{WE}	Workpiece form error	B	Rect.		3,0 µm	3,0	0	0,6	1,80(1)
Combined standard uncertainty, u_c									**3,79**
Expanded uncertainty (k = 2), U									**7,58**
NOTE For an explanation of the indications (1), (2) and (3) concerning the uncertainty components, see B.2.5.5.									

B.2.5.5 First iteration — Discussion of the uncertainty budget

It is documented: $U_{\text{first iteration}}$ = 7,6 µm < target uncertainty U_T = 8 µm.

In Table B.2 there are three large [marked (1)], three mid size [marked (2)] and three small [marked (3)] uncertainty components in the uncertainty of measurement.

The uncertainty components are squared in the formula for combined standard uncertainty. It is therefore difficult to see and understand their influence on u_c. Using instead the variances u^2 gives another and sometimes more understandable picture of the influence of the individual uncertainty components (see Table B.3).

Table B.3 — Influence of the individual uncertainty components on u_c and u_c^2 (25 mm two-point diameter measurement)

Component name	Uncertainty source	Uncert. comp. u_{xx} [µm]	u_{xx}^2 [µm^2]	Percentage of u_c [%]	Percentage of u_c^2 [%]	Uncertainty source
u_{ML} Micrometer — error indication	Measuring equipment	1,80	3,24	23	33	Measuring equipment
u_{MF} Micrometer — flatness 1		0,50	0,25	2		
u_{ML} Micrometer — flatness 2		0,50	0,25	2		
u_{MP} Micrometer — parallelism		1,00	1,00	7		
u_{RR} Repeatability	Operator	1,20	1,44	10	17	Operator
u_{NP} Variation of 0-point		1,00	1,00	7		
u_{TD} Temperature difference	Environment	1,96	3,84	27	27	Environment
u_{TA} Temperature		0,28	0,08	0		
u_{WE} Workpiece form error	Workpiece	1,80	3,24	23	23	Workpiece
Combined standard uncertainty u_c		**3,79**	**14,34**	**100**	**100**	**Total**

From Table B.3, the following can be seen:

— if the external micrometer was without errors at all, U would be reduced from 7,6 µm to 6,2 µm;

— if operator, environment and workpiece were perfect, then U would be reduced from 7,6 µm to 2,2 µm.

It is obvious in this case, that the uncertainty components linked to the measuring process are the dominant components — and not the measuring equipment!

The result U = 7,6 µm, and if the rules of ISO 14253-1 shall comply, then the diameter tolerance of the workpiece shall be reduced 2 × 7,6 µm = 15,2 µm during the production of shafts. This reduction at ∅ 25 mm is equal to the full size of the tolerance IT6 (13 µm).

If U shall only be 10 % of the workpiece tolerance, then the workpiece tolerance shall be IT10 (84 µm). By smaller tolerances, U will be more than 10 % of the tolerance. By IT8 (33 µm), U will be 45 % of the tolerance, and there will be only 10 % of the tolerance left for the production of shafts!

If the target uncertainty shall be 6 µm instead of 8 µm then the uncertainty of measurement from the first iteration is too large (U_{E1} = 7,6 µm). The needed reduction is at least 1,6 µm. This is equal to a reduction of 38 % for u^2.

It will be necessary to look at the most dominant uncertainty component, the temperature difference between workpiece and measuring equipment. It will be possible by changing the procedure and/or measuring the temperatures during production to reduce this 29 % component (29% of u_c^2) to near 0.

An intensive training of the three operators will result in a reduction of the repeatability u_{RR} and the variation between their 0-points (u_{NP}). This will give up to 15 % of the necessary 38 % reduction.

The uncertainty component originating from the form errors of the workpiece is impossible to reduce, when doing only one single measurement of the workpiece. If the number of measurements were increased then this component could be reduced. Doing four measurements and using the mean value will cause a reduction of 20 % of the necessary 38 %. But the effect will be an increase in measuring time! And time is often money!!

In this case there are many ways of reducing the uncertainty of measurement. Which of these to be selected can only be evaluated on the basis of minimizing the costs of a reduction. The costs shall always be the guide of how to reduce the uncertainty of measurement.

In this case a reduction of the components from the micrometer will not be a realistic possibility. The only "equipment solution" is to choose other equipment with smaller (possible) MPE values. This might be an economically sound solution, if the measurement time is also reduced, and it is possible to measure several diameters without influence from the operator.

This could bring down the expanded uncertainty from U = 7,6 µm to 2,6 µm.

B.2.5.6 Conclusion on the first iteration

As illustrated in the example above, the initial setting of the three micrometer MPE values is sufficient to the given target uncertainty and the actual measuring task. The requirements for the micrometer should then be confirmed as:

— Error curve (max. – min.) MPE_{ML} = 6 µm (bilateral specification)

— Flatness of measuring anvils: MPE_{MF} = 1 µm (unilateral specification)

— Parallelism between anvils: MPE_{MP} = 2 µm (unilateral specification)

The micrometer shall comply with these requirements, but reduced with the uncertainties present during the calibration measurements, i.e. U_{SL}, U_{SF} and U_{SP} respectively according to ISO 14253-1 (see clauses B.3, B.4, B.5 and Figure B.1). It is necessary to know the three uncertainties when calibrating the micrometer.

B.2.6 Second iteration

No second iteration is needed in this case. A small decrease of the U value from the first iteration would be possible but no big reduction is possible — as demonstrated — without major changes of the measurement method and procedure.

B.3 Calibration of error of indication of an external micrometer

B.3.1 Requirements

The requirements (MPEs) for the measurement standards (gauge blocks) have not yet been established. These requirements shall be fixed as one of the tasks of the uncertainty budget.

B.3.2 Task and target uncertainty

B.3.2.1 Overall task

The overall task is to measure the range of the error of indication curve. In the error of indication curve there are 11 basic measurements — 11 measurements with a different uncertainty of measurement in the range from 0 mm — 25 mm. To avoid unnecessary uncertainty budgeting work, look for the largest of the 11 uncertainties (25 mm) and see if it is possible to "live" with this uncertainty in the 10 other cases. Try also the smallest (0 mm) as a check.

B.3.2.2 Basic measuring task

To measure the error of indication in 11 positions in the measuring range (0 mm to 25 mm), zero, 2,5, 5, ... 22,5 and 25 mm

B.3.2.3 Target uncertainty for the basic measurements

The target uncertainty for the basic measurements is 1 µm.

B.3.3 Principle, method, procedure and conditions

B.3.3.1 Measurement principle

Measurement of length — Comparison with a known length.

B.3.3.2 Measurement method

The calibration is performed using 10 special gauge blocks with a 2,5 mm module (L = 2,5; 5; ...; 22,5; 25 mm)

B.3.3.3 Initial measurement procedure

— The reading of the external micrometer is compared with the length of a gauge block positioned between the measuring anvils.

— One (calibration) measurement per gauge block. Error of indication:

Error = Micrometer reading – Gauge block length

B.3.3.4 Initial measurement conditions

— The calibration personnel is experienced.

— The room temperature is not controlled.

— A variation over the year in the room is observed to 20 °C ± 8 °C.

— The temperature variation over one hour is less than 0,5 °C.

B.3.4 Graphical illustration of measurement setup

See Figure B.3.

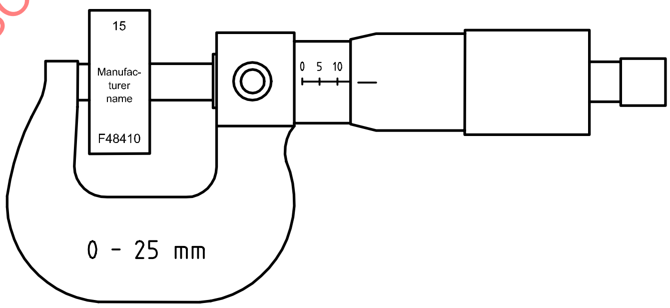


Figure B.3 — Measurement setup

B.3.5 List and discussion of the uncertainty contributors

See Table B.4.

Table B.4 — Overview and comments table for uncertainty components — Calibration of error of indication of a micrometer in the 25 mm measuring point

Designation Low resolution	Detailed designation	Name Uncertainty component	Comments	
u_{SL}		Gauge block length — MPE_{SL}	Requirements for gauge block MPE_{SL} is an unknown variable. Initially gauge block grade 2 (ISO 3650) is chosen.	
u_{RR}	u_{RA}	Resolution	$u_{RA} = \frac{d}{2 \times \sqrt{3}} = \frac{1\ \mu m}{2 \times \sqrt{3}} = 0{,}29\ \mu m$	The largest of the two = u_{RR}
	u_{RE}	Repeatability	An experiment with at least 15 measurements on the same 25 mm gauge block is performed.	
u_{TD}		Temperature difference	Maximum difference observed between the gauge blocks and the micrometer is 1 °C.	
u_{TA}		Temperature	Maximum deviation from standard reference temperature 20 °C is 8 °C.	

B.3.6 First iteration

B.3.6.1 First iteration — Documentation and calculation of the uncertainty components

u_{SL} — Gauge block length **Type B evaluation**

The definitive value of MPE_{SL} has not been fixed yet. It is one of the tasks of the uncertainty budget. Initially gauge blocks of grade 2 are chosen and as MPE_{SL}, the tolerance limit values are taken from ISO 3650. The limit value for a 25 mm gauge block is:

$$a_{SL} = 0{,}6\ \mu m$$

Based on experience from calibration certificates for gauge blocks of the actual make a rectangular distribution is assumed ($b = 0{,}6$):

$$u_{SL} = 0{,}6 \times 0{,}6\ \mu m = 0{,}36\ \mu m$$

u_{RR} — Repeatability/resolution **Type B evaluation**

A repeatability experiment has been made. 15 measurements on a 25 mm gauge block with the actual micrometer. The standard deviation of the experiment is $u_{RE} = 0{,}19\ \mu m$. Therefore the resolution uncertainty component, u_{RA}, shall be chosen as u_{RR} ($u_{RA} > u_{RE}$):

$$u_{RR} = 0{,}29\ \mu m$$

u_{TD} — Temperature difference **Type B evaluation**

The temperature difference between micrometer and gauge blocks is observed to maximum 1 °C. There is no information about which have the highest temperature. Therefore ± 1 °C is assumed. The linear coefficient of thermal expansion, α, is assumed to be 1,1 µm/(100 mm × °C) for the micrometer and the gauge block. The limit value is:

$$a_{TD} = \Delta T \times \alpha \times D = 1\,^{\circ}\mathrm{C} \times 1{,}1 \frac{\mu\mathrm{m}}{100\ \mathrm{mm} \times {}^{\circ}\mathrm{C}} \times 25\ \mathrm{mm} = 0{,}28\ \mu\mathrm{m}$$

A U-distribution is assumed (b = 0,7)

$$u_{TD} = 0{,}28\ \mu\mathrm{m} \times 0{,}7 = 0{,}20\ \mu\mathrm{m}$$

u_{TA} **— Temperature** **Type B evaluation**

The observed maximum difference from standard reference temperature (20 °C) is 8 °C. There is no information about the sign of this deviation, therefore ± 8 °C is assumed. A 10 % maximum difference between the two linear temperature expansion coefficients ($\alpha_{micrometer}$ and $\alpha_{gauge\ block}$) is assumed. The limit value is:

$$a_{TA} = 0{,}1 \times \Delta T_{20} \times \alpha \times D = 0{,}1 \times 8\,^{\circ}\mathrm{C} \times 1{,}1 \frac{\mu\mathrm{m}}{100\ \mathrm{mm} \times {}^{\circ}\mathrm{C}} \times 25\ \mathrm{mm} = 0{,}2\ \mu\mathrm{m}$$

A U-distribution is assumed (b = 0,7)

$$u_{TA} = 0{,}2\ \mu\mathrm{m} \times 0{,}7 = 0{,}14\ \mu\mathrm{m}$$

B.3.6.2 First iteration — Correlation between uncertainty components

It is estimated that no correlation occurs between the uncertainty components.

B.3.6.3 First iteration — Combined and expanded uncertainty

No uncertainty components are correlated. The combined standard deviation is:

$$u_c = \sqrt{u_{SL}{}^2 + u_{RR}{}^2 + u_{TD}{}^2 + u_{TA}{}^2} = 0{,}5\ \mu\mathrm{m}$$

The values from B.3.6.1:

$$u_c = \sqrt{u_{SL}{}^2 + u_{RR}{}^2 + u_{TD}{}^2 + u_{TA}{}^2} = 0{,}5\ \mu\mathrm{m}$$

The expanded uncertainty for the 25 mm measuring point is (coverage factor k = 2):

$$u_{25\ \mathrm{mm}} = 0{,}5\ \mu\mathrm{m} \times 2 = 1{,}0\ \mu\mathrm{m}$$

The expanded uncertainty for the zero-measuring point is:

$$u_{0\ \mathrm{mm}} = 0{,}4\ \mu\mathrm{m} \times 2 = 0{,}8\ \mu\mathrm{m}$$

B.3.6.4 Summary of uncertainty budget — First iteration

See Table B.5.

Table B.5 — Summary of uncertainty budget (first iteration) — Measurement of error of indication (25 mm measuring point)

Component name		Evaluation type	Distri-bution type	Number of measure-ments	Variation limit a^* [µm]	Variation limit a [influence units]	Correlation coefficient	Distribution factor b	Uncer-tainty comp. u_{xx} [µm]
u_{SL}	Gauge block — MPE_{SL}	B	Rect.		0,6 µm	0,6	0	0,6	0,36
u_{RR}	Resolution	B	Rect.		0,5 µm	0,5	0	0,6	0,29
u_{TD}	Temperature difference	B	U		1 °C	0,20	0	0,7	0,20
u_{TA}	Temperature	B	U		8 °C	0,14	0	0,7	0,14
Combined standard uncertainty, u_c									**0,50**
Expanded uncertainty (k = 2), U									**1,00**

B.3.6.5 First iteration — Discussion of the uncertainty budget

The dominant uncertainty components are gauge blocks and resolution. There is no need to reduce the uncertainty of measurement u_C and U in a second iteration. $U < 1$ µm cannot be used because of the resolution 1 µm. Observe that the temperature requirement during calibration is 20 °C ± 8 °C. This temperature range has no significant effect on the uncertainty in this case — short distances! For the larger micrometers this temperature range will result in dominant uncertainty components.

A conservative estimate is to use $U = 1{,}0$ µm for all measuring points between 0 mm and 25 mm. The maximum allowed difference in error of indication during calibration is therefore (see ISO 14253-1):

$$4\ \mu\text{m}\left[\text{MPE}_{ML} - (2 \times U) = 6\ \mu\text{m} - (2 \times 1{,}0\ \mu\text{m}) = 4\ \mu\text{m}\right]$$

B.3.6.6 Conclusion on the first iteration

The target uncertainty criterion is met by the initial assumptions and settings. This fact qualifies grade 2 gauge blocks as measurement standards and qualify the temperature condition of the room: 20 °C ± 8 °C.

B.3.7 Second iteration

No second iteration is needed.

B.4 Calibration of flatness of the measuring anvils

B.4.1 Task and target uncertainty

B.4.1.1 Measuring task

The measuring task consists of measuring the flatness on two ∅ 6 mm measuring anvils of an external micrometer.

B.4.1.2 Target uncertainty

The target uncertainty is 0,15 µm.

B.4.2 Principle, method, procedure and condition

B.4.2.1 Measurement principle

Light interference — Comparison with a flat surface.

B.4.2.2 Measurement method

An optical flat is placed on top of the measuring anvil surface parallel to the general direction of the surface. The number of interference lines is evaluated.

B.4.2.3 Measurement procedure

— An optical flat is wrung to the surface of the measuring anvil.

— The number of interference lines is observed on the nearly symmetrical image [see Figure B.4 b)].

— The deviation from flatness is taken as number of lines times half the wavelength of the monochromatic light used.

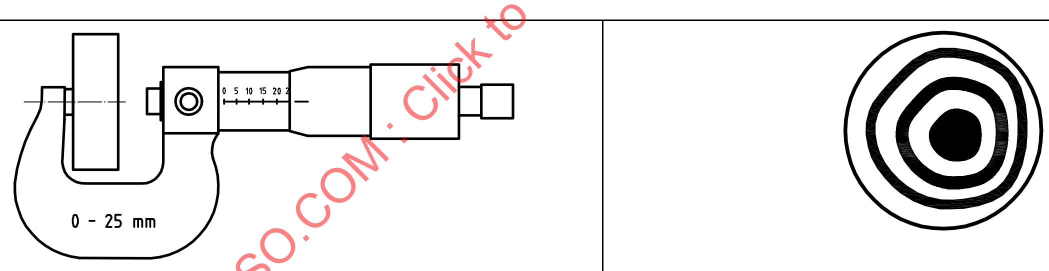
B.4.2.4 Measurement conditions

— No temperature conditions.

— The optical flat shall be acclimatized for at least 1 h.

B.4.3 Graphical illustration of measurement setup

See Figure B.4.



a) Measurement of flatness on the measuring anvils | b) Image to be evaluated

Figure B.4 — Measurement setup

B.4.4 List and discussion of the uncertainty contributors

See Table B.6.

The calibration of flatness of the measuring anvils has only two significant uncertainty components. Flatness of the optical flat and the resolution of reading the interference-image pattern. The optical flat is used in a way, such that the pattern is symmetrical [see Figure B.4 b)].

Table B.6 — Overview and comments table for uncertainty components for calibration of flatness of measuring anvils

Designation Low resolution	Designation High resolution	Name Uncertainty component	Comments
u_{SF}		Flatness — MPE_{SF}	The optical flat is ∅ 31 mm — the flatness is given for this whole area. The area used is only ∅ 6 to ∅ 8 mm.
u_{RR}		Resolution	The resolution is estimated 0,5 × line distance: d = 0,15 µm.

B.4.5 First iteration

B.4.5.1 First iteration — Documentation and calculation of the uncertainty components

u_{SF} — **Flatness of optical flat** **Type B evaluation**

The definitive value of MPE_{SF} is not fixed yet. It is one of the tasks of the uncertainty budget. Initially MPE_{SF} is set to 0,05 µm for a ∅ 8 mm area in the middle of the surface. The limit value:

$$a_{SF} = 0{,}05\ \mu m$$

A rectangular distribution is assumed (b = 0,6):

$$u_{SF} = 0{,}05\ \mu m \times 0{,}6 = 0{,}03\ \mu m$$

u_{RR} — **Resolution** **Type B evaluation**

The wavelength of the light used is assumed to be 0,6 µm. The height difference between the lines of Figure B.4 b) is half a wavelength = 0,3 µm. The resolution is assumed to be:

$$d = 0{,}5 \times \text{line distance} = 0{,}15\ \mu m$$

The uncertainty component u_{RR} (see 8.4.4):

$$u_{RR} = \frac{d}{2} \times 0{,}6 = \frac{0{,}15\ \mu m}{2} \times 0{,}6 = 0{,}05\ \mu m$$

B.4.5.2 First iteration — Correlation between uncertainty components

It is estimated that no correlation occurs between the uncertainty components.

B.4.5.3 First iteration — Combined and expanded uncertainty

$$u_c = \sqrt{u_{SF}^2 + u_{RR}^2}$$

The values from B.4.5.1:

$$u_c = \sqrt{\left(0{,}03^2 + 0{,}05^2\right)\ \mu m^2} = 0{,}06\ \mu m$$

The expanded uncertainty (coverage factor k = 2) is:

$$U = 0{,}06\ \mu m \times 2 = 0{,}12\ \mu m$$

B.4.5.4 Summary of uncertainty budget — First iteration

See Table B.7.

Table B.7 — Summary of uncertainty budget (first iteration) — Calibration of flatness of measuring anvils

Component name	Evaluation type	Distribution type	Number of measurements	Variation limit a^* [influence units]	Variation limit a [µm]	Correlation coefficient	Distribution factor b	Uncertainty comp. u_{xx} [µm]
u_{SF} Flatness of optical flat	B	Rect.		0,05 µm	0,05	0	0,6	0,03
u_{RR} Resolution of interference image	B	Rect.		0,075 µm	0,075	0	0,6	0,05
Combined standard uncertainty, u_c								**0,06**
Expanded uncertainty (k = 2), U								**0,12**

B.4.5.5 First iteration — Discussion of the uncertainty budget

It is obvious that the dominant uncertainty component is the resolution or the reading of the pattern. The flatness deviation of the optical flat is not very important compared with the influence of the resolution. U is in the order of 12 % of the flatness requirement for the measuring anvils of the micrometer MPE_{MF} = 1 µm.

B.4.5.6 Conclusion on the first iteration

The target uncertainty requirement is met. The maximum permissible measured deviation from perfect flatness during calibration is:

$MPE_{MF} - U$ = 1,00 µm – 0,15 µm = 0,85 µm (rule from ISO 14253-1 — unilateral tolerance)

For transformation of the MPE_{SF} ∅ 8 mm requirement to ∅ 30 mm; see clause B.6.

B.4.6 Second iteration

No second iteration is needed.

B.5 Calibration of parallelism of the measuring anvils

B.5.1 Task and target uncertainty

B.5.1.1 Measuring task

The measuring task consists of measuring the parallelism between two ∅ 6 mm measuring anvils of an external micrometer.

B.5.1.2 Target uncertainty

The target uncertainty is 0,30 µm.

B.5.2 Principle, method, procedure and condition

B.5.2.1 Measurement principle

Light interference — Comparison with two parallel surfaces.

B.5.2.2 Measurement method

— An optical parallel is placed between the two measuring anvils and adjusted parallel to one of the anvils.

— The number of interference lines on the other anvil is evaluated.

B.5.2.3 Measurement procedure

— An optical parallel is wrung to the surface of one of the measuring anvils and adjusted to be parallel to the general direction of the surface of the anvil [symmetrical interference image — see Figure B.5 b)].

— The micrometer is "measuring" the optical parallel [see Figure B.5 a)] to bring the measurement force to the right level.

— The number of interference lines is observed on the image on the other anvil [see Figure B.5 c)].

— The deviation from parallelism is taken as number of lines times half the wavelength of the monochromatic light used.

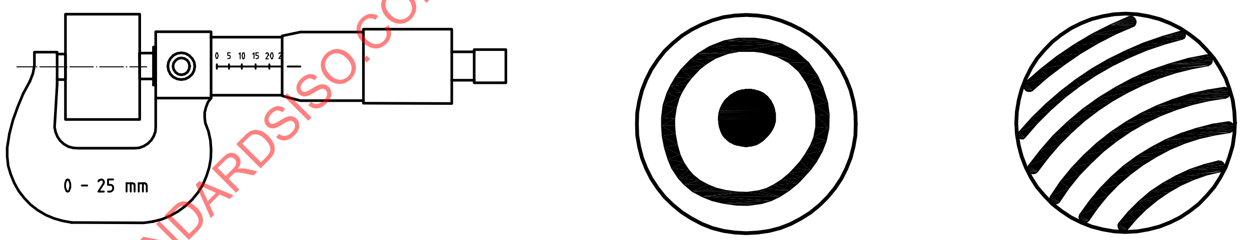
B.5.2.4 Measurement conditions

— No temperature conditions.

— The optical parallel shall be acclimatized for at least 1 h.

B.5.3 Graphical illustration of measurement setup

See Figure B.5.



a) Measurement of parallelism between the measuring anvils

b) Images on the anvils

c) Images on the anvils

Figure B.5 — Measurement setup

B.5.4 List and discussion of the uncertainty contributors

There are three significant uncertainty components in the calibration of the parallelism between the measuring anvils (see Table B.8):

a) the parallelism of the optical parallel;

b) the alignment of the optical parallel to the first measuring anvil;

c) the resolution of reading the interference image pattern on the second measuring anvil.