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Geometrical product specifications (GPS) — Filtration —

Part 20:
Linear profile filters: Basic concepts

Spécification géométrique des produits (GPS) — Filtrage —

Partie 20: Filtres de profil linéaires: Concepts de base

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Foreword

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The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In other circumstances, particularly when there is an urgent market requirement for such documents, a technical committee may decide to publish other types of normative document:

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ISO/TS 16610-20 was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

ISO/TS 16610 consists of the following parts, under the general title *Geometrical product specifications (GPS) — Filtration*:

— *Part 1: Overview and basic concepts*

— *Part 20: Linear profile filters: Basic concepts*

— *Part 22: Linear profile filters: Spline filters*

— *Part 29: Linear profile filters: Spline wavelets*

— *Part 31: Robust profile filters: Gaussian regression filters*

— *Part 32: Robust profile filters: Spline filters*

— *Part 40: Morphological profile filters: Basic concepts*

— *Part 41: Morphological profile filters: Disk and horizontal line-segment filters*

— *Part 49: Morphological profile filters: Scale space techniques*

The following parts are under preparation:

— *Part 21: Linear profile filters: Gaussian filters*

— *Part 26: Linear profile filters: Filtration on nominally orthogonal grid planar data sets*

— *Part 27: Linear profile filters: Filtration on nominally orthogonal grid cylindrical data sets*

— *Part 30: Robust profile filters: Basic concepts*

— *Part 42: Morphological profile filters: Motif filters*

— *Part 60: Linear areal filters: Basic concepts*

— *Part 61: Linear areal filters: Gaussian filters*

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— *Part 71: Robust areal filters: Gaussian regression filters*

— *Part 72: Robust areal filters: Spline filters*

— *Part 80: Morphological areal filters: Basic concepts*

— *Part 81: Morphological areal filters: Sphere and horizontal planar segment filters*

— *Part 82: Morphological areal filters: Motif filters*

— *Part 89: Morphological areal filters: Scale space techniques*

Introduction

This part of ISO/TS 16610 is a geometrical product specification (GPS) Technical Specification and is to be regarded as a global GPS Technical Specification (see ISO/TR 14638). It influences the chain links 3 and 5 of all chains of standards.

For more detailed information about the relation of this part of ISO/TS 16610 to the GPS matrix model, see Annex C.

This part of ISO/TS 16610 develops the basic concepts of linear filters, which include spline filters and spline wavelets, as well as the standardized Gaussian filter (see ISO 11562:1996).

Geometrical product specifications (GPS) — Filtration —

Part 20:
Linear profile filters: Basic concepts

1 Scope

This part of ISO/TS 16610 sets out the basic concepts of linear profile filters.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11562:1996[1)], *Geometrical Product Specifications (GPS) — Surface texture: Profile method — Metrological characteristics of phase correct filters*

ISO/TS 16610-1:2006, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

International vocabulary of basic and general terms in metrology (VIM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 2nd ed., 1993

3 Terms and definitions

For the purposes of this document, the terms and definitions given in VIM and ISO/TS 16610-1, and the following apply.

3.1
linear profile filter
profile filter which separates profiles into long wave and short wave components

3.2
phase correct (linear) profile filter
linear profile filter (3.1) which does not cause phase shifts leading to asymmetrical profile distortions

NOTE Phase correct filters are a particular kind of the so-called linear phase filters, because any linear phase filter can be transformed (simply by shifting its weighting function) to a zero phase filter, which is a phase correct filter.

1) To be replaced by ISO 16610-21.

3.3
weighting function
function for calculating the mean line, which indicates for each point the weight attached by the profile in the vicinity of that point

NOTE The transmission characteristic of the mean line is the Fourier transformation of the weighting function.

3.4
transmission characteristic of a filter
characteristic that indicates the amount by which the amplitude of a sinusoidal profile is attenuated as a function of its wavelength

NOTE The transmission characteristic is the Fourier transformation of the weighting function.

3.5
cut-off wavelength
wavelength of a sinusoidal profile, of which 50 % of the amplitude is transmitted by the profile filter

NOTE 1 Linear profile filters are identified by the filter type and the cut-off wavelength value.

NOTE 2 The cut-off wavelength is the recommended nesting index for linear profile filters.

3.6
filter bank
set of high-pass and low-pass filters, arranged in a specified structure

3.7
multiresolution analysis
decomposition of a profile by a **filter bank** (3.6) into portions of different scales

NOTE The portions at different scales are also referred to as resolutions.

4 Basic concepts

4.1 General

For a filter to conform with this part of ISO/TS 16610, it shall exhibit the characteristics described in 5.1, 5.2, 5.3 and 5.4.

NOTE A concept diagram for linear profile filters is given in Annex A. The relationship to the filtration matrix model is given in Annex B.

The most general linear profile filter is defined by

$$y(x) = \int K(x,\xi) z(\xi) \mathrm{d}\xi \qquad (1)$$

where

$z(\xi)$ is the unfiltered profile;

$y(x)$ is the filtered profile;

$K(x,\xi)$ is a real symmetric and spatial invariant kernel.

If $K(x,\xi) = K(x-\xi)$, the filtering is a convolution

$$y(x) = \int K(x-\xi) z(\xi) \mathrm{d}\xi \qquad (2)$$

and the kernel is called the weighting function of the filter.

However, extracted data are always discrete. Consequently, the filters described here are also discrete. If the weighting function is not discrete (see 4.4, Example 2), it shall be converted into a discrete representation.

4.2 Discrete representation of data

An extracted profile can be represented by a vector. The length n of this vector is equal to the number of data points. The sampling is assumed to be uniform, i.e. the sampling interval is constant. The ith data point of the profile is therefore the ith component of the vector.

$$(a_1 a_2 \dots a_i \dots a_{n-1} a_n) \tag{3}$$

4.3 Discrete representation of the linear profile filter

A linear profile filter is represented by a square matrix. The dimension of this matrix is equal to the number of data points to be filtered. If the filter is non-periodic, the matrix is a constant diagonal (Toeplitz) matrix:

$$\begin{pmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & & & & \\ & c' & b' & a & b & c & & & \\ & & c' & b' & a & b & c & & \\ & & & c' & b' & a & b & c & \\ & & & & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \tag{4}$$

Otherwise, if the filter is periodic, the matrix is a circulant matrix:

$$\begin{pmatrix} a & b & c & \dots & \dots & c' & b' \\ b' & a & b & c & \dots & \dots & c' \\ c' & b' & a & b & c & \dots & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & \dots & c' & b' & a & b & c \\ c & \dots & \dots & c' & b' & a & b \\ b & c & \dots & \dots & c' & b' & a \end{pmatrix} \tag{5}$$

If the filter is phase correct, the matrix representing the filter is symmetrical, i.e. $b = b', c = c', \dots$ (generally $a_{ij} = a_{ji}$) is valid. The sum of the matrix elements a_{ij} of each row i is constant, and for low-pass filters equals one, i.e.

$$\sum_j a_{ij} = 1 \tag{6}$$

NOTE In the case of a symmetrical matrix, the sum of the matrix elements a_{ij} of each column j is also constant, and equals one, i.e. $\sum_i a_{ij} = 1$ is valid, too.

4.4 Discrete representation of the weighting function

Given that each row of the matrix representation of the filter is identical after being shifted accordingly, the matrix elements may be represented by one single row. i.e.

$$a_{ij} = s_k \text{ with } k = i - j \tag{7}$$

The values s_k form a vector s of a dimension equal to the length of the input or output data vector, respectively. This vector is the discrete representation of the weighting function of the filter.

NOTE 1 The length of the weighting function is usually much smaller than the length of the data set. In this case, s contains zeros at each end.

EXAMPLE 1 The moving average filter is frequently used for easy smoothing of a data set, which is not necessarily an optimal method. In the following example of a filter with a discrete weighting function, where a length of three has been taken, the weighting function is given by

$$\left(...0,0,\frac{1}{3},\frac{1}{3},\frac{1}{3},0,0...\right) \tag{8}$$

NOTE 2 The weighting function is often also called the impulse response function, because it is the output data set of the filter, if the input data set is only a single unity impulse $(...0,0,0,1,0,0,0...)$.

If the weighting function is given as a continuous function, it shall be sampled in order to obtain a discrete data set. The sampling interval used shall be equal to the sampling interval of the extracted data. It is mandatory to re-normalize the sampled data of the weighting function subsequently, in order to fulfil the condition that they shall sum to unity, thus avoiding bias effects (for details concerning bias effects, see [2]).

EXAMPLE 2 The Gaussian filter in accordance with ISO 11562:1996 is an example of a continuous weighting function $s(x)$ defined by the equation

$$s(x)=\frac{1}{\alpha\lambda_c}\exp\left[-\pi\left(\frac{x}{\alpha\lambda_c}\right)^2\right] \tag{9}$$

where

x is the distance from the centre (maximum) of the weighting function;

λ_c is the cut-off wavelength;

α is a constant given by the following equation:

$$\alpha=\sqrt{\frac{\log 2}{\pi}}=0{,}469\,7... \tag{10}$$

The graph of this weighting function is shown in Figure 1.

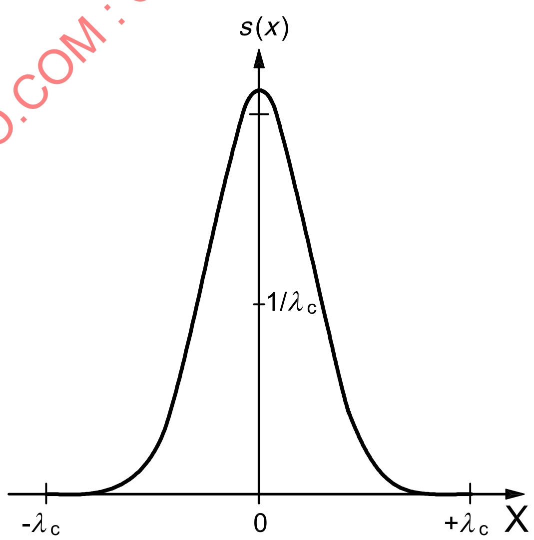


Figure 1 — Example of a continuous weighting function (Gaussian filter)

The sampled data s_k of the weighting function after a re-normalization are given by

$$s_k = \frac{1}{C}\exp\left[-\pi\left(\frac{\Delta x}{\alpha\lambda_c}\right)^2 k^2\right] \tag{11}$$

with the sampling interval Δx, and the normalization constant

$$C = \sum_k \exp\left[-\pi\left(\frac{\Delta x}{\alpha\lambda_c}\right)^2 k^2\right] \tag{12}$$

5 Linear profile filters

5.1 Filter equations

If the filter is represented by the matrix S, the input data by the vector z and the output data by the vector w, then the filtering process is described by the linear equation

$$w = Sz \tag{13}$$

This equation is the filter equation. If S^{-1} is the inverse matrix of the filter matrix S, then

$$z = S^{-1}w \tag{14}$$

is a valid filter equation, too.

NOTE 1 The filter can be defined by the matrix S or by the inverse matrix S^{-1}, whichever leads to a simpler definition. However, the weighting function is only given by the rows of the matrix S.

NOTE 2 The inverse matrix sometimes does not exist, in which case the filtering process is not invertible, i.e. data reconstruction is impossible. Such a filter is called unstable. The stability of a filter can be seen from its transfer function (see 5.3). An unstable filter has a transfer function $H(\omega)$, which is zero for at least one frequency ω.

EXAMPLE The matrix of the moving average filter mentioned above

$$\frac{1}{3}\begin{pmatrix} \ddots & \ddots & \ddots & & & \\ & 1 & 1 & 1 & & \\ & & 1 & 1 & 1 & \\ & & & 1 & 1 & 1 \\ & & & \ddots & \ddots & \ddots \end{pmatrix} \tag{15}$$

is not invertible, and therefore the filter is unstable. If the filter is changed to a moving average filter $(\alpha < 1/2)$

$$\frac{1}{1+2\alpha}\begin{pmatrix} \ddots & \ddots & \ddots & & & \\ & \alpha & 1 & \alpha & & \\ & & \alpha & 1 & \alpha & \\ & & & \alpha & 1 & \alpha \\ & & & \ddots & \ddots & \ddots \end{pmatrix} \tag{16}$$

it becomes stable.

The inverse matrix S^{-1} is a constant diagonal matrix or a circulant matrix, if S is, respectively, a constant diagonal matrix or a circulant matrix. The inverse matrix S^{-1} is symmetrical, if S is symmetrical.

5.2 Discrete convolution

The filter equation can be written as

$$w_i = \sum a_{ij} z_j = \sum s_{i-j} z_j \quad (17)$$

The latter expression is known as a discrete convolution with the abbreviated notation $w = s * z$. If the filter matrix is circulant, the convolution is circular, i.e. the coefficients s_{i-j} shall be seen as being extended periodically at both ends (wrapped around).

NOTE The circular convolution can be calculated by using the fast Fourier transformation (FFT), which is often faster than the usual convolution.

EXAMPLE An example of a discrete convolution is shown in Figure 2. Here the filtered value w_i for $i = 3$ is calculated by multiplication of the data values at the points $j = 0 \ldots 6$, with the sampled values of the weighting function at the points $i - j$, and a subsequent summation.

$$w_i = \sum_{j=0}^{n} s_{i-j} z_j \quad (18)$$

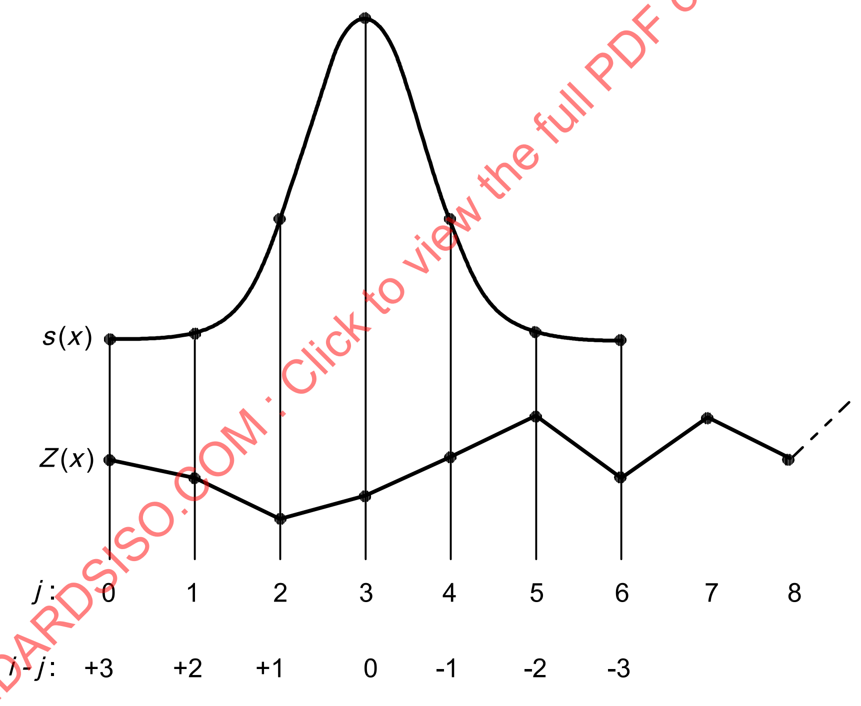


Figure 2 — Example of a discrete convolution

5.3 Transfer function

Taking the discrete Fourier transformation of the discrete convolution yields

$$W = HZ \tag{19}$$

where

W is the discrete Fourier transform of the output vector w;

H is the discrete Fourier transform of the discrete representation of the weighting function s;

Z is the discrete Fourier transform of the input vector z.

The function H is called the transfer function of the filter. It depends on the wavelength λ or the angular frequency $\omega = 2\pi / \lambda$, as the Fourier transformation transforms to the wavelength or frequency domain.

The Fourier transformation $H(\omega)$ of the discrete representation of the weighting function by the vector s with components s_k is calculated by

$$H(\omega) = \sum_k s_k \mathrm{e}^{-i\omega k} = s_0 + \sum_{k \neq 0} s_k \left(\cos\omega k + i \sin\omega k\right) \tag{20}$$

Generally speaking, the transfer function turns out to be complex valued. However, if the weighting function is symmetrical, i.e. $s_{-k} = s_k$ is valid, the formula is simplified to

$$H(\omega) = s_0 + 2\sum_{k>0} s_k \cos\omega k \tag{21}$$

leading to a real transfer function.

For a phase correct filter, the transfer function is always a real function, i.e. the imaginary part is zero. This is due to the fact that the imaginary part represents a phase shift, which is not permitted for phase correct filters.

EXAMPLE 1 The transfer function for the moving average filter mentioned above is

$$H(\omega) = \frac{1 + 2\cos\omega}{3} \tag{22}$$

The graph of this transfer function is shown in Figure 3. This filter is not stable, because $H(\omega) = 0$ if $\omega = \pm 2\pi/3$.

The moving average filter shown in Figure 3 is a low-pass filter, because $H(\omega)$ has its highest values around the frequency $\omega = 0$. By contrast, for a high-pass filter, $H(\omega)$ would have its highest values in the high frequency region near $\omega = \pm\pi$. If a low-pass filter transfer function $H(\omega)$ is given, the simplest way to get a high-pass filter transfer function $H(\omega)$ is to calculate $H_0(\omega) = 1 - H_0(\omega)$. However, this is not always the best possible choice.

EXAMPLE 2 The high-pass filter is complimentary to a moving average filter, and is therefore stable. The (weighted) moving average filter has the (low-pass) transfer function

$$H_0(\omega) = \frac{1 + 2\alpha\cos\omega}{1 + 2\alpha} \tag{23}$$

The high-pass filter then has the transfer function

$$H_1(\omega) = 1 - H_0(\omega) = \frac{2\alpha}{1 + 2\alpha}(1 - \cos\omega) \tag{24}$$

Both transfer functions are shown in Figure 4.

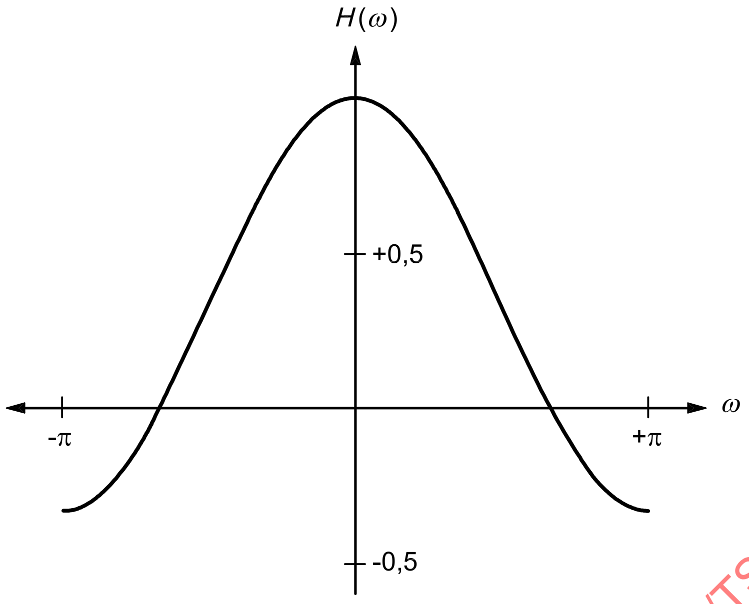
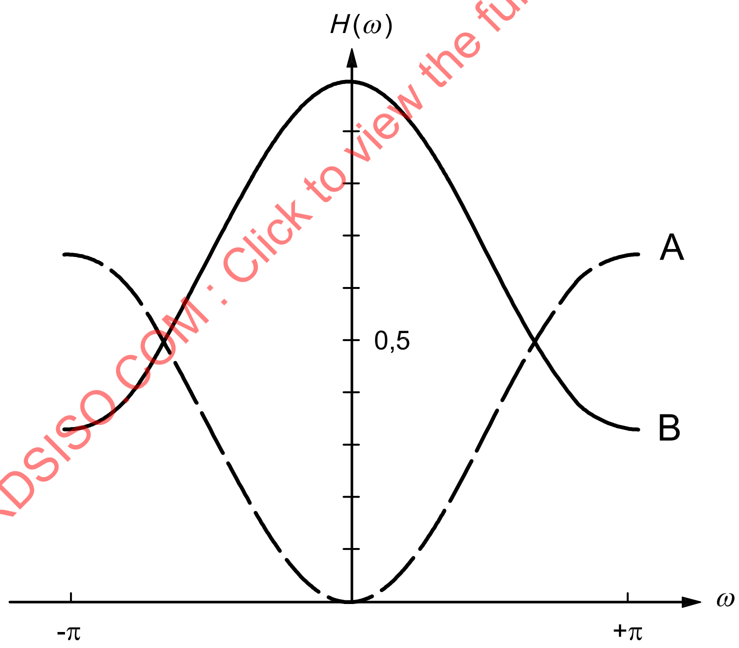


Figure 3 — Transfer function of the moving average filter of length three



Key

A high-pass transfer function

B low-pass transfer function

Figure 4 — Low-pass and high-pass transfer function with $\alpha = 0,25$

The weighting function of the low-pass filter is

$$\left(...0,0,\frac{\alpha}{1+2\alpha},\frac{1}{1+2\alpha},\frac{\alpha}{1+2\alpha},0,0...\right) \tag{25}$$

The weighting function of the high-pass filter can easily be shown to be

$$\left(...0,0,-\frac{\alpha}{1+2\alpha},\frac{2\alpha}{1+2\alpha},-\frac{\alpha}{1+2\alpha},0,0...\right) \tag{26}$$

This filter is called a (weighted) moving difference filter.

5.4 Filter banks

In a two-channel filter bank, the two filters are normally a high-pass and a low-pass filters. These are indicated by their transfer functions $H_0(\omega)$ and $H_1(\omega)$. The object is to separate the input data into a low frequency (long wavelength) component and a high frequency (short wavelength) component.

EXAMPLE: In terms of roughness, the profile $z(x)$ is separated into a waviness component $w(x)$ and a roughness component $r(x)$ by a low-pass filter $H_0(\omega)$ and a high-pass filter $H_1(\omega)$ (see Figure 5).

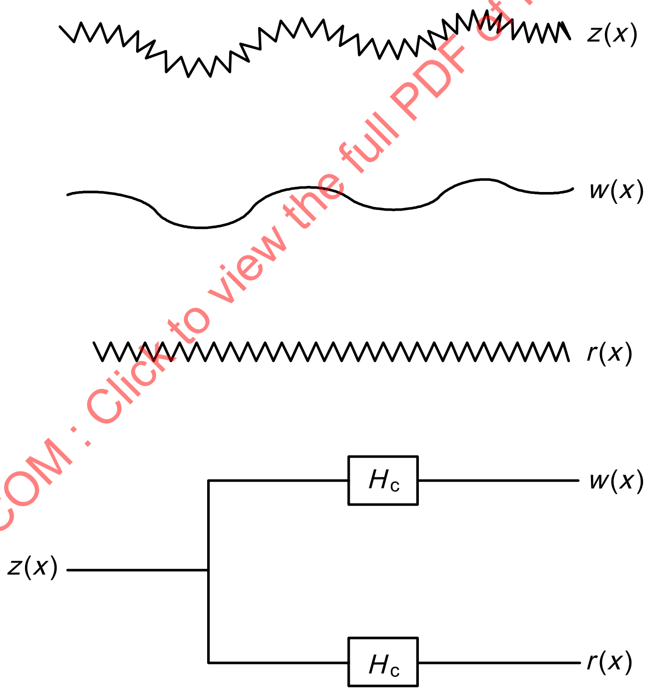


Figure 5 — Separation of a measured profile into a waviness component $w(x)$ and a roughness component $r(x)$ by a low-pass filter and a high-pass filter

Generally speaking, the transfer functions of the low-pass and high-pass filters overlap (with non zero values at the same wavelength, as in Figure 4). This cannot be avoided in the practical implementation of the filter. The separation is not ideal, because input data portions whose frequencies fall within the overlap region go partly to both channels. This results in aliasing in each channel. Any reconstruction of the input data from the filtered data needs to take this fact into account.

Cascading of filter banks leads to multiresolution analysis. Each filtering stage gives finer details of the profile data. They appear at multiple scales. However, filter banks shall be specifically designed to achieve multiresolution.

Annex A
(informative)

Concept diagram

The following is a concept diagram for this part of ISO/TS 16610.