AEROSPACE INFORMATION REPORT

AIR1657™ REV. C

Issued 1981-01
Revised 2021-05

Superseding AIR1657B

(R) Handbook of Hydraulic Metric Calculations

RATIONALE

AIR1657 has been updated to Revision C for the following reasons:

- To incorporate technical updates and corrections.
- To include editorial and pictorial improvements.

FOREWORD

Hydraulic calculations may be conducted directly in the International System (SI) metric units. Calculations in SI metric units could not be simpler. Data, however, is not always in these units and conversions are necessary. It is generally preferable to convert the data and then proceed with calculations in the SI metric units.

The terminology is defined in each section where it is required to avoid any misunderstanding of dual uses of symbols such as N = Newton or N = rpm.

There can be confusion where symbols for a physical quantity have the same letters, (for example, mN (milli newton) and Newton meter (Nm)). To prevent this confusion, where the first letter is a multiple of an SI unit, there should be no spaces between the two letters (mN). If the two letters are combinations of separate units, then there should be a space or a dot between the two letters. In this document, a space is employed (for example N m).

Care should be taken when converting from one unit to a higher or lower power of the same measurement, for example, 2.0 mm is 2.0 x 10^{-3} m, not 2.0 x 10^{3} m.

TABLE OF CONTENTS

1. SCOPE

This SAE Aerospace Information Report (AIR) provides details of how to perform hydraulic system calculations using equations that incorporate the metric International System of Units (SI).

2. REFERENCES

2.1 Applicable Documents

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or +1 724-776-4970 (outside USA), www.sae.org.

AIR1362 Aerospace Hydraulic Fluids Physical Properties

ARP4386 Terminology and Definitions for Aerospace Fluid Power, Actuation and Control Technologies

AS1241 Fire Resistant Phosphate Ester Fluid for Aircraft

2.1.2 ASTM Publications

Available from ASTM International, 100 Barr Harbor Drive, P.O. Box C700, West Conshohocken, PA 19428-2959, Tel: 610-832-9585, www.astm.org.

ASTM SI 10 American National Standard for Use of the International System of Units (SI): The Modern Metric System

2.1.3 ICAO Publications

Available from International Civil Aviation Organization, 999 University Street, Montreal, Quebec H3C 5H7, Canada, Tel: +1 514-954-8219, http://www.icao.int/.

Doc 7488-CD Manual of the ICAO Standard Atmosphere: extended to 80 kilometres (262 500 feet)

2.1.4 NAS Publications

Available from Aerospace Industries Association, 1000 Wilson Boulevard, Suite 1700, Arlington, VA 22209-3928, Tel: 703-358-1000, www.aia-aerospace.org.

NAS 10001 Preferred Metric Units for Aerospace

2.1.5 U.S. Government Publications

Copies of these documents are available online at https://quicksearch.dla.mil.

MIL-PRF-5606 Hydraulic Fluid, Petroleum Base; Aircraft, Missile and Ordinance, NATO Code 515 (Inactive for new design)

MIL-PRF-83282 Hydraulic Fluid, Fire Resistant, Synthetic Hydrocarbon Base, Aircraft, NATO Code H537

MIL-PRF-87257 Hydraulic Fluid, Fire Resistant, Low Temperature, Synthetic Hydrocarbon Base, Aircraft and Missile, NATO Code 538

2.2 Related Publications

The following publications are provided for information purposes only and are not a required part of this SAE Technical Report.

2.2.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or 724-776-4970 (outside USA), www.sae.org.

TSB 003 Rules for SAE Use of SI (Metric) Units

2.2.2 ISO Publications

Copies of these documents are available online at http://webstore.ansi.org/.

ISO 80000-1 Quantities and units — Part 1: General

2.3 Definitions

FLUID SHEAR STRESS: This is the shear stress within a fluid while it is flowing which varies with the velocity gradient across a given sheared section.

NOTE: Hydraulic fluids can be considered as Newtonian fluids, in which the shear stress is proportional to the rate of shear.

HYDROSTATIC PRESSURE: This is the pressure that is statically generated by the weight of a column of fluid.

RATED SYSTEM PRESSURE: This is the nominal steady-state pressure that is generated by the hydraulic power generation system.

NOTE: Rated System Pressure is equivalent to the Design Operating Pressure (DOP) for commercial aircraft and System Pressure for military aircraft.

SPECIFIC GRAVITY: This is the ratio of the density of a substance to the density of a given reference material.

NOTE: For the purpose of this document, the substance is a hydraulic fluid and water at 15 °C and at sea level is the given reference material.

Refer to ARP4386 for the other definitions and terms.

3. DEFINITION OF METRIC UNITS

The SI system is coherent because force, pressure, and power are expressed in terms of basic quantities of mass, length, and time without empirical constants. SI engineering units have been devised to be scaled to each domain to facilitate calculations, handling and criticizing of numerical results because having a size 0.1 to 1000 suits the way the human brain processes them.

Kilogram (kg), meter (m), and second (s) are the basic units. Unity (1) is the coefficient of the basic units in their definition. Multiples of the basic units are defined by powers of 10^3 as shown in Table 1.

NOTE: kg is now only a unit of mass (not weight), weight is a force in N.

Table 1 - Recommended multiples of SI units

10^9	1000000000	giga	G
10^6	1000000	mega	M
10^3	1000	kilo	k
1	1	basic unit	basic unit
10^{-3}	0.001	milli	m
10^{-6}	0.000001	micro	µ
10^{-9}	0.000000001	nano	n

These multiples provide usefulness and convenience for practical purposes. Multiples other than powers of 10^3 are obsolete as shown in Table 2.

Table 2 - Not recommended multiples of units

10^2	100	hecto	h
10^1	10	deca	da
10^{-1}	0.1	deci	d
10^{-2}	0.01	centi	c

The SI metric system was established by the International Organization for Standardization (ISO) and supersedes the CGS metric system, where centimeter (cm), and gram (g) and second (s) were the basic units and the earlier metric system where kilogram force was the basic unit.

NOTE: If it is not mentioned in this document, to conform to SAE policy:

- Pressure is in kPa, not Mpa.
- Gallons are U.S. gallons.

Table 3 provides the conversion of metric units. NAS 10001 and ASTM SI 10 are available for conversion of metric and English units.

Table 3 - Metric units

Physical Quantity	Symbol	Dimension	SI (Metric) Unit	Other SI Unit	Conversion to SI Units
Mass	M	M	kg, kilogram	g, gram	10^{-3} kg
Length	l	L	m, meter	cm, centimeter mm, millimeter	10^{-2} m 10^{-3} m
Time	t	T	s, second	min, minute h, hour	60 s 3600 s
Angle	α	-	rad, radian	rev, revolution degree	2π rad π/180 rad
Absolute Temperature	T	-	K, Kelvin		
Temperature	T	-	°C, degree Celsius (centigrade)		K - 273.2
Frequency	f	T^{-1}	Hz, Hertz	cps, cycle/s	1 Hz
Angular frequency	ω	T^{-1}	Hz, Hertz	Rad/s, radian/s	1 Hz
Area	A	L^2	m^2	mm^2 cm^2	10-6 m^2 10-4 m^2
Volume	V	L^3	m^3	L, liter mL cm^3, cc mm^3, μL	10-3 m^3 10-6 m^3 10-6 m^3 10-9 m^3
Mass Density	ρ	ML^{-3}	kg/m^3	kg/L g/cm^3	10-3 kg/m^3 10-3 kg/m3
Moment of Inertia	J	ML^2	kg m^2	g cm^2	
Velocity	v	LT^{-1}	m/s	km/s km/h	103 m/s 103/3600 m/s
Acceleration	a	LT^{-2}	m/s^2	g	9.81 m/s^2
Force	F	MLT^{-2}	N, Newton	kgf (force) kN, kilo N daN	9.81 N 103 N 10 N
Work, Energy, Heat	F x l	ML^2T^{-2}	J, Joule	W·h kW·h N·m	3600 J 3600 kJ 1 J
Pressure	P	$ML^{-1}T^{-2}$	Pa, Pascal	N/m^2 kPa Mpa bar mm Hg	1 Pa 103 Pa 106 Pa 105 Pa 133.3 Pa
Flow, Volumetric	Qv	L^3T^{-1}	m^3/s	L/s L/min	10^{-3} m^3/s 10^{-3}/60 m^3/s
Torque	T	ML^2T^{-2}	N·m, Newton·meter	kN m	10^3 N m
Speed of Rotation	ω	T^{-1}	rad/s	rpm, rev/minute	2π/60 rad/s
Power, Heat Flow	W Q_h	ML^2T^{-3}	W, Watt	kW mW	10^3 W 10^{-3} W
Spring Rate	F/l	MT^{-2}	N/m	kN/m kN/mm daN/mm	10^3 N/m 10^6 N/m 10^4 N/m
Surface Tension	ɣ	MT^{-2}	N/m	mN/m	10^{-3} N/m
Bulk Modulus	B	$ML^{-1}T^{-2}$	Pa, Pascal	kPa Mpa Gpa	10^3 Pa 10^6 Pa 10^9 Pa
Dynamic Viscosity	μ	$ML^{-1}T^{-1}$	Pa s	mPa s centipoise Poiseuille	10^{-3} Pa s 10^{-3} Pa s 1 Pa·s

Table 3 - Metric units (continued)

Physical Quantity	Symbol	Dimension	SI (Metric) Unit	Other SI Unit	Conversion to SI Units
Kinematic Viscosity	ν	L^2T^{-1}	m^2/s	mm^2/s centistoke, cSt	10^{-6} m^2/s 10^{-6} m^2/s
Specific Heat	C_p	L^2T^{-2}	J/(kg K)	kJ/(kg K)	10^3 J/(kg K)
Thermal Conductivity	k	MLT^{-3}	W/(m K)		
Surface Thermal Conductance	h_c	MT^{-3}	W/(m^2 K)	kW/(m^2 K)	10^3 W/(m^2 K)
Heat Capacity Rate	C	ML^2T^{-3}	W/K watt/K	kW/K mW/K	10^3 W/K 10^{-3} W/K

Table 4 provides factors for conversion from English units to SI Metric units.

Table 4 - Factors for converting from English units to SI Metric units

Physical Quantity	Symbol	Dimension	English Unit	SI Metric Unit	English Conversion to Metric
Mass	M	M	lb_m slug	kg kg	0.4536 kg 14.594 kg
Length	l	L	nautical mile mile ft, foot in, inch μin, microinch	km km m mm μm	1.852 km 1.609 km 0.3048 m 25.4 mm 0.0254 μm
Time	t	T	s, second min, minute h, hour	s	1 s 60 s 3600 s
Angle	α	-	rev, revolution degree	rad, radian	2π rad π/180 rad
Absolute Temperature	T	-	°R, degree Rankine	K, Kelvin	K/1.8
Temperature	T	-	°F, degree Fahrenheit	C, Celsius	(F-32)/1.8
Frequency	f	T^{-1}	cps, cycle/s	Hz, hertz	1 Hz
Area	A	L^2	ft^2 ft^2 in^2	m^2 cm^2 mm^2	0.0929 m^2 929 cm^2 645 mm^2
Volume	V	L^3	ft^3 U.S. gallon Imperial gallon in^3 drop	L, liter L L mL mm^3	28.32 L 3.785 L 4.546 L 16.39 mL 50 mm^3
Mass Density	ρ	ML^{-3}	lbm/gal lbm/ft^3	kg/L kg/m^3	0.1198 kg/L 16.02 kg/m^3
Moment of Inertia	J	ML^2	lbm·ft^2 lbm·in^2 lbf·in·s^2	Kg m^2 Kg mm^2 kg m^2	0.04214 kg m^2 292.6 kg mm^2 0.113 kg m^2
Velocity	v	LT^{-1}	ft/s in/s knot mile/hour	m/s mm/s km/h km/h	0.3048 m/s 25.4 m/s 1.852 km/h 1.609 km/h
Acceleration	a	LT^{-2}	ft/s^2	m/s^2	0.3048 m/s^1
Force	F	MLT-2	lbf kip, kilopound	N, Newton kN	4.448 N 4.448 kN

Table 4 - Factors for converting from English units to SI Metric units (continued)

Physical Quantity	Symbol	Dimension	English Unit	SI Metric Unit	English Conversion to Metric
Work, Energy, Heat	F x l	ML^2T^{-2}	ft·lb$_f$ BTU, British Thermal Unit	J kJ	1.356 J 1.055 kJ
Pressure	P	$ML^{-1}T^{-2}$	lb$_f$/in^2, psi ksi, kilopound per in^2 in Hg	kPa kpa kPa	6.895 kPa 6895 kPa 3.386 kPa
Flow, Volumetric	Q_v	L^3T^{-1}	gal/min, gpm in^3/s, cis	L/min L/min	3.785 L/min 983.4 mL/min
Torque	T	ML^2T^{-2}	lb$_f$·ft lb$_f$·in oz$_f$·in	N m N m N m	1.356 N m 0.1130 N m 0.0071 N m
Speed of Rotation	ω	T^{-1}	rpm, rev/minute	rad/s	2π/60 rad/s
Power, Heat Flow	W Q_h	ML^2T^{-3}	ft·lb$_f$ /s hp kBTU/h	W kW kW	1.356 W 0.7457 kW 0.293 kW
Spring Rate	F/l	MT^{-2}	lb$_f$ /in kip/in	N/m kN/m	175.1 N/m 175.1 kN/m
Surface Tension	ɣ	MT^{-2}	lbf /in grainf/in	N/m mN/m	175.1 N/m 25 mN/m
Bulk Modulus	B	$ML^{-1}T^{-2}$	kip/in^2, ksi	kPa	6895 kPa
Dynamic Viscosity	μ	$ML^{-1}T^{-1}$	lbf ·s/ft^2 10-3 lb·s/ft^2	Pa s mPa s centipoise	47.88 Pa s 47.88 mPa s 47.88 cP
Kinematic Viscosity	v	L^2T^{-1}	ft^2/s ft^2/s	m^2/s mm^2/s	0.09259 m^2/s 926 mm^2/s
Specific Heat	Cp	L^2T^{-2}	BTU/(lbm·F)	kJ/(kg K)	4.187 kJ/(kg K)
Thermal Conductivity	k	MLT^{-3}	BTU/(h·ft·F)	W/(m K)	1.731 W/(m K)
Surface Thermal Conductance	hc	MT^{-3}	BTU/(h·ft^2·F)	kW/(m^2 K)	5.679 W/(m^2 K)
Heat Capacity Rate	C	ML^2T^{-3}	BTU/(h·F)	W/K	0.527 W/K

4. USE OF METRIC UNITS IN HYDRAULIC SYSTEM ANALYSIS - HYDROSTATICS

4.1.1 Primary Units

The hydrostatic pressure (P) is by definition:

$$P = \rho g h \quad \text{(Eq. 1A)}$$

$$(ML^{-1}T^{-2} = ML^{-3} \times LT^{-2} \times L) \quad \text{(Eq. 1B)}$$

where:

P = pressure, (Pa) - ($ML1^{-1}T^{-2}$)

ρ = mass density, (kg/m^3) - (ML^{-3})

g = gravitational acceleration, (m/s^2) - (LT^{-2})

$g = 9.81\ m/s^2$ at sea level, 45 degree latitude

h = static pressure head, (m) - (L)

4.1.1.1 Practical Units

By the definition of liter (L):

$$1\ L = 1\ m^3/1000 \quad \text{(Eq. 2)}$$

Therefore:

$$\rho = \text{mass density (kg/L)} \quad \text{(Eq. 3A)}$$

$$1\ kg/L = 1000\ kg/m^3 \quad \text{(Eq. 3B)}$$

The hydrostatic pressure in practical units is:

$$P = \rho g h \quad \text{(Eq. 4)}$$

where:

P = pressure, (kPa) - (1 kPa = 1000 Pa)(10^3 Pa, $ML^{-1}T^{-2}$)

ρ = mass density, (kg/L) - ($10^3\ kg/m^3$, ML^{-3})(1 kg/L = 1000 kg/m^3)

g = gravitational acceleration, (m/s^2) - (LT^{-2}) = 9.81 m/s^2 at sea level, 45 degree latitude

h = head, (m) - (L)

4.1.1.2 Application - Suction Pressure

Given:

a. MIL-PRF-5606 hydraulic fluid at 20 °C.

b. The hydraulic reservoir is at 2.27 m above the baseline.

c. An actuator is at 5.32 m above the baseline and is directly connected to the reservoir via the return connection of the controlling valve.

The hydraulic reservoir is open to the sea-level atmosphere during maintenance.

Calculate:

1. The suction pressure (P) that the actuator seals should be able to withstand without air ingestion.

2. The absolute pressure (P) at the actuator.

The density of MIL-PRF-5606 at 20 °C and 0 kPa per Figure 1 is:

$$\rho = 0.859\ kg/L \quad \text{(Eq. 5)}$$

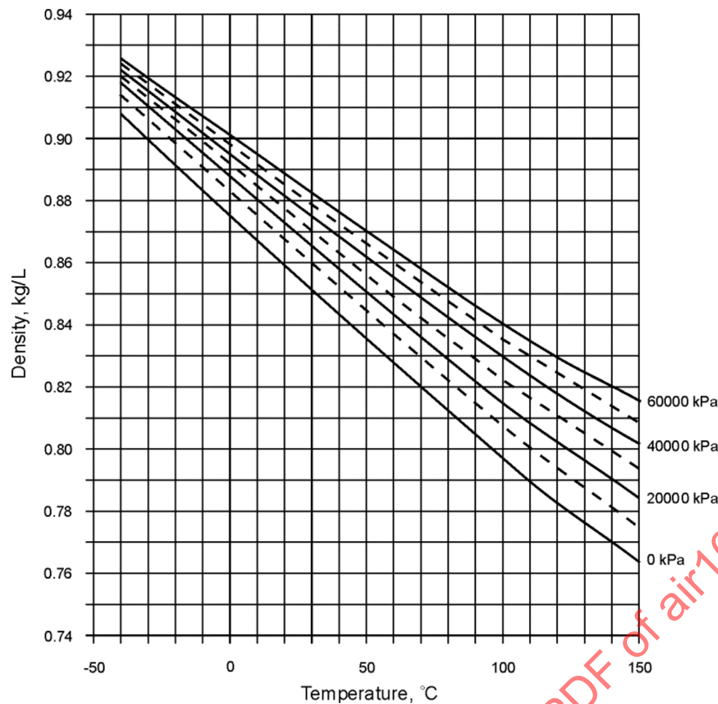

Figure 1 - Mass density of MIL-PRF-5606

The difference in elevation is:

$$h = 5.32 - 2.27 = 3.05 \text{ m} \quad \text{(Eq. 6)}$$

The suction pressure (below atmospheric) in kilopascals (kPa):

$$P = 0.859 \times 9.81 \times 3.05 \quad \text{(Eq. 7A)}$$

$$P = 25.7 \text{ kPa} \quad \text{(Eq. 7B)}$$

The standard sea-level atmospheric pressure is 101.3 kPa per ICAO Doc 7488-CD.

The absolute pressure at the actuator is:

$$101.3 - 25.7 = 75.6 \text{ kPa} \quad \text{(Eq. 8)}$$

4.2 Hydrostatic Axial Force

4.2.1 Primary Units

The axial force is:

$$F = A \times P \quad \text{(Eq. 9A)}$$

$$(MLT^{-2} = L^2 \times ML^{-1}T^{-2}) \quad \text{(Eq. 9B)}$$

where:

F = force, (Newton, N) - (MLT^{-2})

A = area, (m^2) - (L^2)

P = pressure, (Pascal, Pa) - ($ML^{-1}T^{-2}$)

4.2.2 Practical Units

F = force, (Newton, N) - (MLT^{-2})

A = area, (mm^2) - (10^{-6} m^2, L^2)

P = pressure, (kPa) - (10^3 Pa, $ML^{-1}T^{-2}$)

4.2.3 Application - Hydrostatic Test

A single-acting hydraulic cylinder of 50.8 mm inside diameter is proof pressure tested to 1.5 factor of the rated system pressure, 20690 kPa.

Calculate the axial force (F) on the cylinder cap on the full area side.

The test pressure is:

$$P = 1.5 \text{ x rated system pressure} \quad \text{(Eq. 10A)}$$

$$P = 1.5 \text{ x } 20690 \text{ kPa} \quad \text{(Eq. 10B)}$$

$$P = 31035 \text{ kPa} \quad \text{(Eq. 10C)}$$

The cap area is:

$$A = \pi \text{ x } (50.8)^2/4 \quad \text{(Eq. 11A)}$$

$$A = 2027 \text{ mm}^2 \quad \text{(Eq. 11B)}$$

The force is:

$$F = 31035 \text{ x} 10^3 \text{ Pa x } 2027 \text{x} 10^{-6} \text{ m}^2 \quad \text{(Eq. 12A)}$$

$$F = 62908 \text{ N} \quad \text{(Eq. 12B)}$$

$$F = 62.9 \text{ kN} \quad \text{(Eq. 12C)}$$

5. USE OF METRIC UNITS IN HYDRAULIC SYSTEM ANALYSIS - HYDRODYNAMICS

5.1 Bernoulli's Theorem

5.1.1 Total Head

The total head in steady state flow of an incompressible fluid is:

$$H = h + v^2 / 2g + z \quad \text{(Eq. 13A)}$$

$$(L = L + (LT^{-1})^2 / LT^{-2} + L) \quad \text{(Eq. 13B)}$$

where:

h = static head, (m) - (L)

NOTE: Head "loss" can be either negative or positive if energy is introduced from outside the considered fluid domain.

v = velocity of flow, (m/s) - (LT^{-1})

g = gravitational acceleration, (m/s^2) - (LT^{-2})

z = elevation of section above datum, (m) - (L)

Bernoulli's theorem states that total head (H) along a streamline is constant, provided that the head losses (DH) between sections are accounted for:

$$H_1 = H_2 + DH \qquad \text{(Eq. 14A)}$$

$$h_1 + v_1^2 / 2g + z_1 = h_2 + v_2^2 / 2g + z_2 + DH \qquad \text{(Eq. 14B)}$$

where:

Subscripts 1 and 2 refer to sections

DH = loss in total head, (m) - (L)

5.1.2 Velocity Head

By definition, the velocity head (h_v) is:

$$h_v = v^2 / 2g \qquad \text{(Eq. 15A)}$$

$$(L = (LT^{-1})^2 / LT^{-2}) \qquad \text{(Eq. 15B)}$$

5.1.3 Application - Liquid Column Head

MIL-PRF-5606 fluid at 50 °C and 20690 kPa pressure is flowing in a tube at a velocity of 3 m/s. What is the velocity head in terms of:

a. Liquid column height?

b. Pressure?

1. In terms of liquid column height:

$$h_v = v^2 / 2g \qquad \text{(Eq. 16)}$$

where:

v = 3.0 m/s - (LT^{-1})

g = 9.81 m/s^2 - (LT^{-2}) at sea level, 45 degree latitude

Then:

$$h_v = 3.0^2 / (2 \times 9.81) \qquad \text{(Eq. 17A)}$$

$$h_v = 0.46 \text{ m} \qquad \text{(Eq. 17B)}$$

2. In terms of pressure head (P_v) (dynamic pressure):

$$P_v = \rho\, g\, h_v \qquad \text{(Eq. 18A)}$$

$$(ML^{-1}T^{-2} = ML^{-3}LT^{-2}L) \qquad \text{(Eq. 18B)}$$

$$P_v = (\rho\ g) \times (v^2 / 2g) = \rho\ v^2 / 2 \quad \text{(Eq. 18C)}$$

$$(ML^{-1}T^{-2} = ML^{-3}(LT^{-1})^2) \quad \text{(Eq. 18D)}$$

where:

ρ = 0.85 kg/L - (ML^{-3}) per Figure 1

v = 3 m/s - (LT^{-1})

Then:

$$P_v = 0.85 \times 3^2 / 2 \quad \text{(Eq. 19A)}$$

$$P_v = 3.83\ \text{kPa} \quad \text{(Eq. 19B)}$$

5.2 Viscosity

5.2.1 Dynamic Viscosity

The dynamic viscosity is:

$$\mu = (F / A) / (d_V/d_y) \quad \text{(Eq. 20A)}$$

$$(ML^{-1}T^{-1} = (MLT^{-2}/L^2) / (LT^{-1}/L)) \quad \text{(Eq. 20B)}$$

In primary units:

μ = dynamic viscosity, (Pa·s) - ($ML^{-1}T^{-1}$)

F = force (N), Newton - (MLT^{-2})

A = area (m^2) - (L^2)

d_V = variation of velocity, (m/s) - (LT^{-1})

dy = normal distance, (m) - (L)

In practical units:

μ= dynamic viscosity, (centipoise) - ($ML^{-1}T^{-1}$)

NOTE: 1 centipoise = 1 mPa s

F = force (N), Newton - (MLT^{-2})

A = area, (mm^2) - (10^{-6} m^2, L^2)

d_V = variation of velocity, (m/s) - (LT^{-1})

d_y = normal distance, (mm) - (10^{-3} m, L)

5.2.2 Kinematic Viscosity

It is by definition, the shear force per unit area and per unit mass density due to a unit velocity gradient normal to the flow:

$$\nu = (F / (A\ \rho)) / (d_V/d_y) \quad \text{(Eq. 21A)}$$

$$(L^2T^{-1} = (MLT^{-2} / (L^2 \times ML^{-3})) / (LT^{-1}/L)) \quad \text{(Eq. 21B)}$$

That is:

$$\nu = \mu / \rho \quad \text{(Eq. 22A)}$$

$$(L^2T^{-1} = ML^{-1}T^{-1} / ML^{-3}) \quad \text{(Eq. 22B)}$$

In basic units:

ν = kinematic viscosity, (m^2/s) - (L^2T^{-1})

F = force (N), Newton - (MLT^{-2})

A = area, (m^2) - (L^2)

ρ = mass density, (kg/m^3) - (ML^{-3})

d_V = variation of velocity, (m/s) - (LT^{-1})

d_y= normal distance, (m) - (L)

In practical units:

ν = kinematic viscosity, (mm^2/s) - (10^{-6} m^2 /s, L^2T^{-1})

F = force, (Newton) - (MLT^{-2})

A = area, (mm^2) - (10^{-6} m^2, L^2)

ρ = mass density, (kg/liter) - (ML^{-3})

d_V = variation of velocity, (m/s) - (LT^{-1})

d_y = normal distance, (mm) - (10^{-3} m, L)

See Table 3 where 1 mm^2/s = 1 centistoke.

5.2.3 Fluid Shear Stress

Fluid shear stress (P) is by definition:

$$\tau = \mu \, v / \delta \quad \text{(Eq. 23A)}$$

$$(ML^{-1}T^{-2} = ML^{-1}T^{-1} \times LT^{-1}/L) \quad \text{(Eq. 23B)}$$

where:

τ = shear stress, (Pa) - ($ML^{-1}T^{-2}$)

μ = absolute viscosity, (mPa s) - ($ML^{-1}T^{-1}$)

v = velocity, (m/s) - (LT^{-1})

δ = radial clearance, (m) - (L)

5.2.4 Examples

5.2.4.1 Absolute Viscosity

Calculate the absolute viscosity (μ) of MIL-PRF-83282 hydraulic fluid at -30 °C and 10000 kPa by solving Equation 24:

$$\mu = \nu \rho \quad \text{(Eq. 24)}$$

Viscosity ν = 1400 mm^2/s (centistokes) obtained from Figure 2.

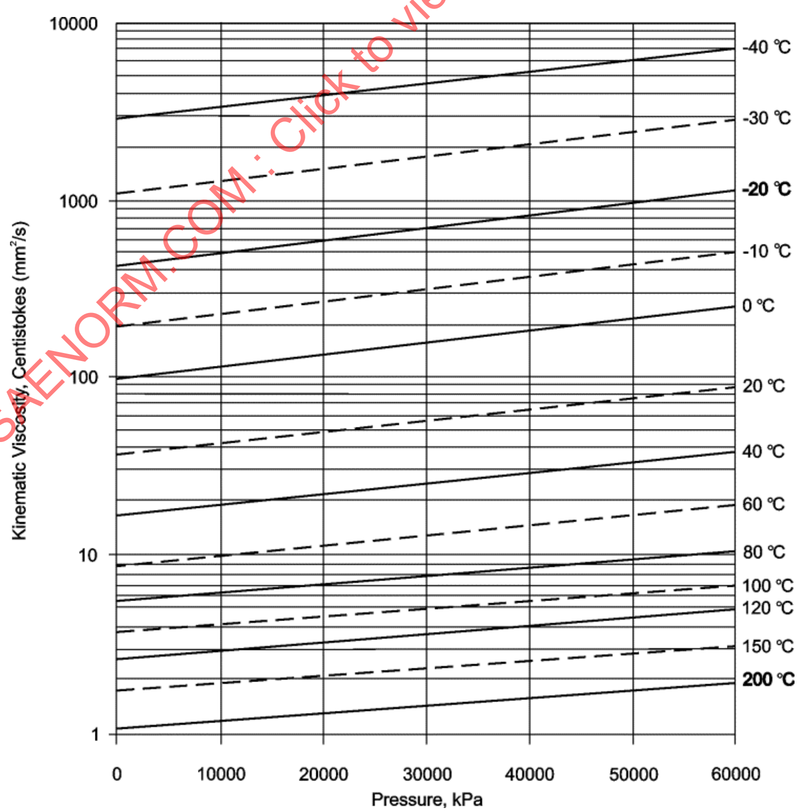

Figure 2 - Kinematic viscosity versus pressure, MIL-PRF-83282

Density ρ = 0.88 kg/L obtained from Figure 3.

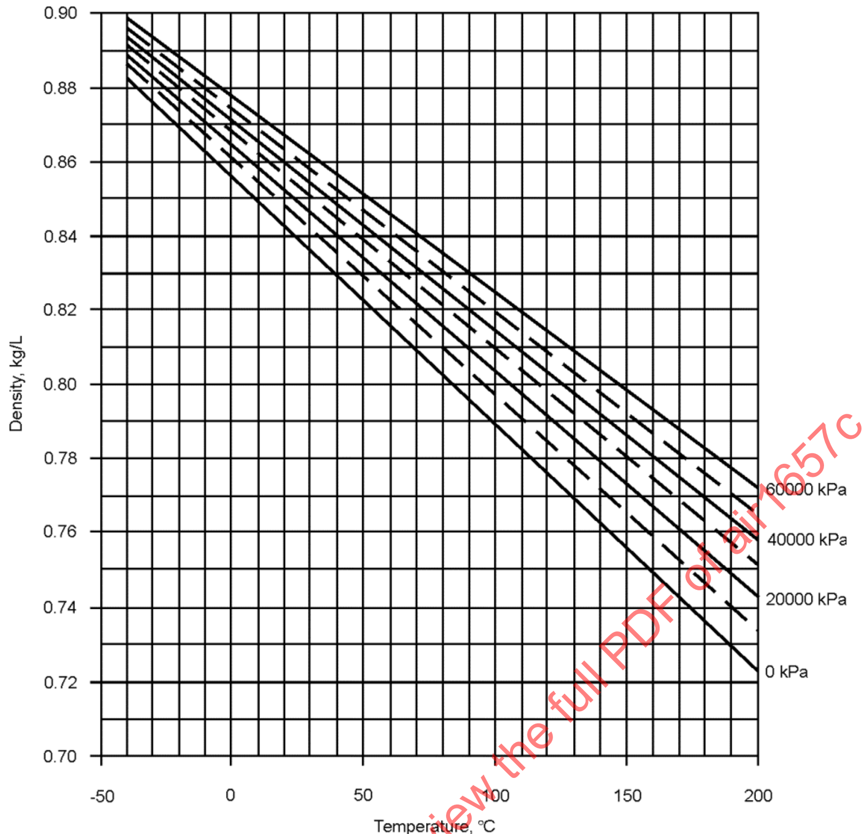

Figure 3 - Mass density of MIL-PRF-83282

In primary units:

$$\mu = 1400 \times 10^{-6}\ m^2/s \times 880\ kg/m^3 = 1.23\ Pa\ s \quad \text{(Eq. 25)}$$

In practical units:

$$\mu = 1400\ mm^2/s \times 0.88\ kg/L = 1230\ mPa\ s \quad \text{(Eq. 26)}$$

In the usual (CGS) units:

$$\mu = 1400\ cst \times 0.88\ g/cc = 1230\ centipoise \quad \text{(Eq. 27A)}$$

$$\mu = 1230\ mPa\ s \quad \text{(Eq. 27B)}$$

5.2.4.2 Application - Direct Drive Valve

Calculate the fluid shear stress (τ) between a valve slide and its sleeve using MIL-PRF-83282 hydraulic fluid at -30 °C and 10000 kPa.

The slide is stroked 0.6 mm in 8 ms at a constant speed.

The diametral clearance is 3.8 μm (150 μin).

The slide is assumed to be centered.

Calculate the velocity (v) and the distance (δ) between valve slide and sleeve at the solid/fluid interface:

$$v = 0.6 \times 10^{-3} \text{ m} / 8 \times 10^{-3} \text{ s} = 0.075 \text{ m/s} = 75 \text{ mm/s} \quad \text{(Eq. 28)}$$

$$\delta = 3.8\ \mu\text{m} / 2 = 1.9\ \mu\text{m} \quad \text{(Eq. 29)}$$

Use Equation 23:

$$\tau = \mu\, v / \delta \quad \text{(Eq. 23)}$$

where:

μ = absolute viscosity = 1230 mPa s - (10^{-3} Pa s, $ML^{-1}T^{-1}$) from 5.2.4.1

v = velocity = 75 mm/s - (10^{-3} m/s, LT^{-1})

δ = radial clearance =1.9 µm - (10^{-6} m, L)

In primary units:

$$\tau = 1.23 \text{ Pa s} \times 0.075 \text{ m/s} / 1.9 \times 10^{-6} \text{ m} = 48.6 \times 10^{3} \text{ Pa} \quad \text{(Eq. 30A)}$$

In practical units:

$$\tau = 1230 \text{ mPa s} \times 0.075 \text{ m/s} / 1.9\ \mu\text{m} = 48.6 \text{ kPa} \quad \text{(Eq. 30B)}$$

$$\tau = 48.6 \text{ kPa} \quad \text{(Eq. 30C)}$$

Calculate the axial force (F) and the power (W) required to overcome viscous shear of a valve slide of 6 mm diameter by 20 mm total combined lap length.

Calculate the shear area:

$$A = \pi \times 6 \text{ mm} \times 20 \text{ mm} = 377 \text{ mm}^2 \quad \text{(Eq. 31)}$$

The axial force (F) is:

$$F = \tau\, A \quad \text{(Eq. 32A)}$$

$$(MLT^{-2} = ML^{-1}T^{-2} \times L^2) \quad \text{(Eq. 32B)}$$

where:

τ = 48.6 kPa - (10^3 Pa, $ML^{-1}T^{-2}$)

A = 377 mm^2 - (10^{-6} m^2, L^2)

$$F = 48.6 \times 10^{3} \text{ Pa} \times 377 \times 10^{-6} \text{ m}^2 = 18.3 \text{ N } (MLT^{-2}) \quad \text{(Eq. 33)}$$

The power (W) is:

$$W = F\ v \qquad \text{(Eq. 34A)}$$

$$(ML^2T^{-3} = MLT^{-2} \times LT^{-1}) \qquad \text{(Eq. 34B)}$$

where:

F = 18.3 N (MLT^{-2})

v = 0.075 m/s (LT^{-1})

$$W = 18.3\ \text{N} \times 0.075\ \text{m/s} = 1.37\ \text{W}\ (ML^2T^{-3}) \qquad \text{(Eq. 35)}$$

5.3 Reynolds Number (Re)

Reynolds Number (Re) is:

$$Re = \rho\ v\ D / \mu \qquad \text{(Eq. 36A)}$$

$$(ML^{-3} \times LT^{-1} \times L / ML^{-1}T^{-1}) \qquad \text{(Eq. 36B)}$$

where:

ρ = mass density, (kg/m^3) - (ML^{-3})

v = mean velocity of flow, (m/s) - (LT^{-1})

D = inside diameter (or its equivalent) of tubing, (m) - (L)

μ = dynamic viscosity, (Pa s) - ($ML^{-1}T^{-1}$)

Using the kinematic viscosity ν, Reynolds Number is simply:

$$Re = v\ D / \nu \qquad \text{(Eq. 37A)}$$

$$(LT^{-1} \times L / L^2T^{-1}) \qquad \text{(Eq. 37B)}$$

where:

v = mean velocity of flow, (m/s) - (LT^{-1})

D = inside diameter (or its equivalent) of tubing, (m) - (L)

ν = kinematic viscosity, (m^2/s) (- L^2T^{-1})

Flow is generally laminar if Re is less than 2000.

Flow is always turbulent if Re is more than 4000.

Upstream conditions determine the flow transitioning between laminar and turbulent conditions. In general, conservative design practices assume the highest pressure drop condition, with the transition typically occurring at Re = 1190.

NOTE: Assume the flow is turbulent when the pressure loss has a functional effect (for example, the sizing of pipelines) and the flow is in the transition phase.

5.3.1 Practical Units

The tubing inside diameter is usually given in millimeters (mm).

The kinematic viscosity is usually given in centistokes.

According to Table 3:

$$1 \text{ cst} = 10^{-6} \text{ m}^2/\text{s} \quad \text{(Eq. 38A)}$$

$$1 \text{ cst} = 1 \text{ mm}^2/\text{s} \quad \text{(Eq. 38B)}$$

Let the velocity also be in millimeter per second (mm/s) units for consistency.

Then:

$$Re = v\, D / \nu \quad \text{(Eq. 39)}$$

If the velocity is in m/s, instead of mm/s, then:

$$Re = 1000\, v\, D / \nu \quad \text{(Eq. 40)}$$

where:

v = velocity, (m/s) - (LT^{-1})

D = diameter, (mm) - (10^{-3} m, L)

ν = centistoke, (mm^2/s) - (10^{-6} m^2/s, L^2T^{-1})

5.3.2 Application - Laminar, Turbulent Flow

A pump has an 18.85 mL/rev displacement and rotates at 6000 rpm speed. The pump is supplied with MIL-PRF-5606 fluid at a pressure of 350 kPa through a suction line with 31.75 mm OD and 0.89 mm wall thickness.

What is the Reynolds number (Re) in the suction line at -45 °C (-49 °F) and 90 °C (194 °F)?

When is the flow laminar or turbulent?

The pump suction flow rate is:

$$Q = (18.85 \text{ mL/rev}) \times (6\,000 \text{ rpm} / 60 \text{ s/min}) \quad \text{(Eq. 41A)}$$

$$Q = 1\,885 \text{ mL/s} \quad \text{(Eq. 41B)}$$

$$Q = 1\,885 \text{ mL/s} \times (60 \text{ s/min}) / (1\,000 \text{ mL/L}) \quad \text{(Eq. 41C)}$$

$$Q = 113 \text{ L/min} \quad \text{(Eq. 41D)}$$

The inside diameter of the tube is:

$$D = OD - 2t \quad \text{(Eq. 42A)}$$

$$D = 31.75 \text{ mm} - 1.78 \text{ mm} \quad \text{(Eq. 42B)}$$

$$D = 29.97 \text{ mm} \quad \text{(Eq. 42C)}$$

The open cross sectional area is:

$$A = \pi/4 \times (D)^2 \quad \text{(Eq. 43A)}$$

$$A = 705.5 \text{ mm}^2 \quad \text{(Eq. 43B)}$$

The velocity of flow is:

$$v = Q / A \quad \text{(Eq. 44)}$$

Where all the data are in consistent mm units:

v = velocity, (mm/s) - (10^{-3} m/s, LT^{-1})

Q = flow, (mm^3/s) - (10^{-9} m^3/s, L^3T^{-1})

A = area, (mm^2) - (10^{-6} m^2, L^2)

Remarking that:

$$1 \text{ mL} = 10^3 \text{ mm}^3 \quad \text{(Eq. 45)}$$

Then v in mm/s units is:

$$v = (Q / A) \times 10^3 \quad \text{(Eq. 46)}$$

Then:

$$v = (1885 \text{ mL/s} / 705.5 \text{ mm}^2) \times 10^3 \quad \text{(Eq. 47A)}$$

$$v = 2672 \text{ mm/s} \quad \text{(Eq. 47B)}$$

$$v = 2672 \text{ mm/s} / (1000 \text{ mm/m}) \quad \text{(Eq. 47C)}$$

$$v = 2.67 \text{ m/s} \quad \text{(Eq. 47D)}$$

The kinematic viscosity of MIL-PRF-5606 fluid is read from Figure 4:

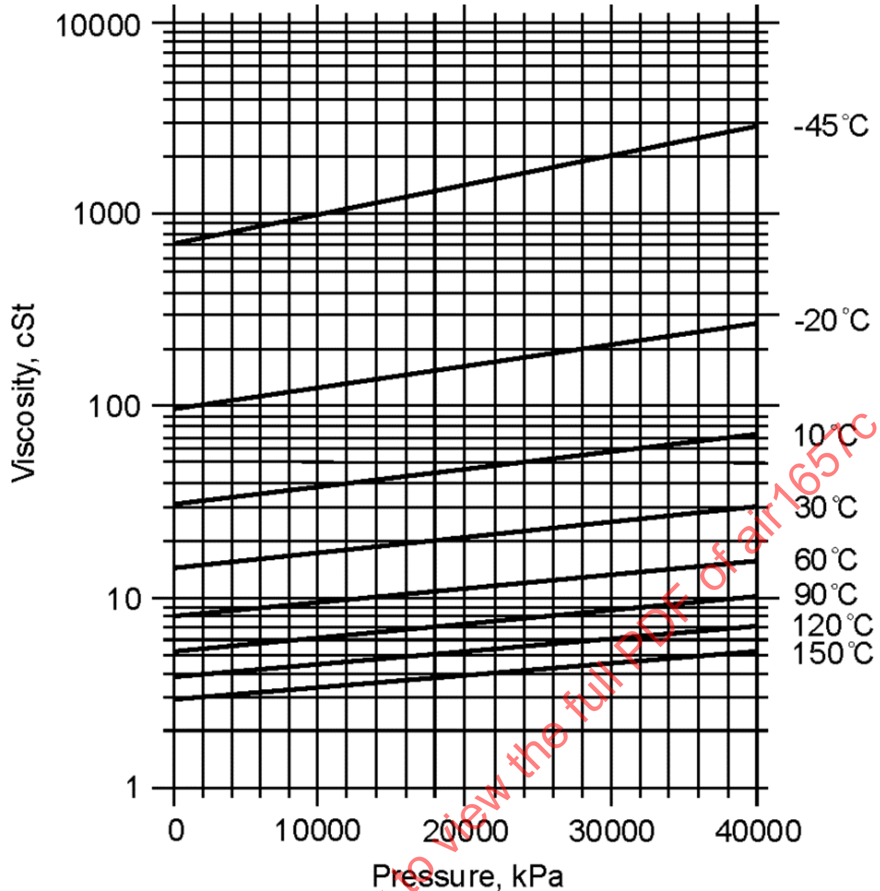

Figure 4 - Kinematic viscosity of MIL-PRF-5606

$$\nu = 700\ mm^2/s \text{ at } -45\ °C \quad \text{(Eq. 48A)}$$

$$\nu = 5.2\ mm^2/s \text{ at } 90\ °C \quad \text{(Eq. 48B)}$$

At -45 °C:

$$Re = 2672\ mm/s \times 29.97\ mm / 700\ mm^2/s \quad \text{(Eq. 49A)}$$

$$Re = 114 \quad \text{(Eq. 49B)}$$

Therefore, flow is laminar because Re < 2000.

At 90 °C:

$$Re = 2672\ mm/s \times 29.97\ mm / 5.2\ mm^2/s \quad \text{(Eq. 50A)}$$

$$Re = 15.4 \times 10^3 \quad \text{(Eq. 50B)}$$

Therefore, flow is turbulent because Re > 4000.

5.4 Friction Loss

5.4.1 Darcy's Equation

Head loss in a tube through wall friction is:

$$\Delta H = f\ (L/D)\ v^2 / 2g \quad \text{(Eq. 51A)}$$

$$((L/L) \times (LT^{-1})^2 / LT^{-2}) \quad \text{(Eq. 51B)}$$

where:

ΔH = head of fluid column, (m) - (L)

f = friction factor, (dimensionless)

L/D = length to diameter ratio, (dimensionless) - (L/L)

$v^2/2g$ = velocity head, (m) - (L)

NOTES:

1. Concerning the friction factor symbol, "λ" may be used instead of "f".
2. The f friction factor used in this document refers to the Darcy-Weisbach friction factor f. It should not be confused with the Fanning friction factor f, which is 4 times lower than the Darcy-Weisbach for the λ friction factor.

5.4.2 Pressure Drop

Pressure drop is given by:

$$\Delta P = f\ (L/D)\ \rho\ v^2 / 2 \quad \text{(Eq. 52A)}$$

$$((L/L) \times (ML^{-3}) \times (LT^{-1})^2) \quad \text{(Eq. 52B)}$$

In primary units:

ΔP = total pressure drop, (Pa) - ($ML^{-1}T^{-2}$)

L = length, (m) - (L)

D = inside diameter, (m) - (L)

ρ = density, (kg/m^3) - (ML^{-3})

v = velocity, (m/s) - (LT^{-1})

By definition:

$$1\ kPa = 10^3\ Pa \text{ and } 1\ kg/L = 10^3\ kg/m^3 \quad \text{(Eq. 53)}$$

In practical units:

ΔP = total pressure drop, (kPa) (10^3 Pa, $ML^{-1}T^{-2}$)

L = length, (mm) - (10^{-3} m, L)

D = inside diameter, (mm) - (10^{-3} m, L)

ρ = density, (kg/liter) - (kg/m^3, ML^{-3})

v = velocity, (m/s) - (LT^{-1})

5.4.3 Friction Factor - Laminar and Turbulent Flow

Friction factor in laminar flow is given by:

$$f = 64 / Re \quad \text{(Eq. 54)}$$

Friction factor for turbulent flow in smooth tubing is approximately:

$$f = 0.316 / Re^{0.25} \quad \text{(Eq. 55)}$$

NOTE: Assume the flow is turbulent when the pressure loss has a functional effect (for example, the sizing of pipelines) and the flow is in the transition phase.

5.4.4 Application - Head Loss

What is the head loss (ΔP) in a 4.57 m long, 30 mm ID tube carrying 113 L/min of MIL-PRF-5606 fluid at -40 °C and 100 °C temperatures and at atmospheric pressure?

$$L / D = 4570 \text{ mm} / 30 \text{ mm} = 152.3 \quad \text{(Eq. 56)}$$

From Figure 1, density is obtained:

$$\rho = 0.908 \text{ kg/L at -40 °C} \quad \text{(Eq. 57A)}$$

$$\rho = 0.796 \text{ kg/L at 100 °C} \quad \text{(Eq. 57B)}$$

From the 5.3.2 example, velocity is:

$$v = 2.67 \text{ m/s} \quad \text{(Eq. 58)}$$

5.4.4.1 Dynamic Pressure:

$$\rho v^2/2 = 3.24 \text{ kPa at -40 °C} \quad \text{(Eq. 59A)}$$

$$\rho v^2/2 = 2.84 \text{ kPa at 100 °C} \quad \text{(Eq. 59B)}$$

From the 5.3.2 example, Reynolds Number:

$$Re = 182 \text{ at -40 °C (Laminar)} \quad \text{(Eq. 60A)}$$

$$Re = 16300 \text{ at 100 °C (Turbulent)} \quad \text{(Eq. 60B)}$$

Friction factor, using the applicable equation:

a. -40 °C, laminar flow:

$$f = 64 / 182 \quad \text{(Eq. 61A)}$$

$$f = 0.352 \quad \text{(Eq. 61B)}$$

b. 100 °C, turbulent flow

$$f = 0.316 / (16\ 300)^{0.25} \quad \text{(Eq. 62A)}$$

$$f = 0.028 \quad \text{(Eq. 62B)}$$

5.4.4.1.1 Pressure Drop:

a. -40 °C:

$$\Delta P = 0.352 \times 152.3 \times 3.24 \text{ kPa} \quad \text{(Eq. 63A)}$$

$$\Delta P = 174 \text{ kPa} \quad \text{(Eq. 63B)}$$

b. 100 °C:

$$\Delta P = 0.028 \times 152.3 \times 2.84 \text{ kPa} \quad \text{(Eq. 64A)}$$

$$\Delta P = 12.1 \text{ kPa} \quad \text{(Eq. 64B)}$$

5.4.4.2 Momentum

By definition,

$$\text{momentum} = M\,v \quad \text{(Eq. 65A)}$$

$$(MLT^{-1} = M(LT^{-1})) \quad \text{(Eq. 65B)}$$

where:

M = mass, (kg) - (M)

v = velocity, (m/s) - (LT^{-1})

The force required to change the momentum is, by Newton's law:

$$F = d(M_v)/dt \quad \text{(Eq. 66A)}$$

$$(MLT^{-2} = MLT^{-1}/T) \quad \text{(Eq. 66B)}$$

where:

F = external force, (Newton) - (MLT^{-2})

$d(M_v)/dt$ = rate of change of momentum - (MLT^{-1}/T)

The momentum change of a column of fluid flowing through a tube:

$$d(M_v)/dt = M\,(dv/dt) + v\,(dM/dt) \quad \text{(Eq. 67)}$$

where:

dv/dt = acceleration of the fluid, (m/s^2) - (LT^{-1}/T)

dM/dt = mass flow rate, (kg/s) - (MT^{-1})

In steady state flow:

$$dv/dt = 0 \quad \text{(Eq. 68A)}$$

$$d(Mv)/dt = v\ dM/dt \quad \text{(Eq. 68B)}$$

In an incompressible fluid:

$$dM/dt = \rho\ Q \quad \text{(Eq. 69A)}$$

$$(MT^{-1} = ML^{-3} \times L^{3}T^{-1}) \quad \text{(Eq. 69B)}$$

where:

ρ = density, (kg/m^3) - (ML^{-3})

Q = flow rate, (m^3/s) - (L^3T1^{-1})

Practical Units

$$F = \rho\ Q\ v \quad \text{(Eq. 70)}$$

where:

F = force, (Newton) - (MLT^{-2})

ρ = density, (kg/L) - (10^3kg/m^3, ML^{-3})

Q = flow rate, (L/s) - (L^3T^{-1})

ρ Q = mass flow rate, (kg/s) - (MT^{-1})

v = velocity (m/s) - (LT^{-1})

5.4.5 Application - Servovalve Nozzle

A nozzle of 0.20 mm inside diameter is supplied with MIL-PRF-5606 fluid at 50 °C. The pressure upstream of the nozzle is 10000 kPa and the downstream pressure is the atmosphere.

Calculate the jet velocity, flow rate, mass flow and momentum.

Bernoulli's Equation with negligible losses through the nozzle yields:

$$P = \rho v^2/2 \quad \text{(Eq. 71A)}$$

$$(ML^{-1}T^{-2} = ML^{-3} \times (LT^{-1})2) \quad \text{(Eq. 71B)}$$

Solving for velocity:

$$v = (2P/\rho)^{0.5} \quad \text{(Eq. 72A)}$$

$$(LT^{-1} = ML^{-1}T^{-2}/ML^{-3})^{0.5}) \quad \text{(Eq. 72B)}$$

where:

ρ = 0.843 kg/L per Figure 1 - (ML^{-3})

P = 10000 kPa - (10^3 Pa, $ML^{-1}T^{-2}$)

Therefore:

$$v = (2 \times 10^4 / 0.843)^{0.5} \quad \text{(Eq. 73A)}$$

$$v = 154 \text{ m/s} \quad \text{(Eq. 73B)}$$

$$(LT^{-1}) \quad \text{(Eq. 73C)}$$

Flow rate is:

$$Q = A\ v \quad \text{(Eq. 74A)}$$

$$(L^3T^{-1} = L^2 \times LT^{-1}) \quad \text{(Eq. 74B)}$$

where:

$A = \pi\ D^2/4$, - (L^2)

$A = \pi\ (0.20 \text{ mm})^2/4$, $(10^{-6} \text{ m}^2, L^2)$

$A = 0.0314 \text{ mm}^2$ $(10^{-6} \text{ m}^2, L^2)$

Note that:

$$1 \text{ mL} = 10^3 \text{ mm}^3 \quad \text{(Eq. 75)}$$

Therefore:

$$Q = 0.031\,4 \text{ mm}^2 \times 154 \text{ m/s} \quad \text{(Eq. 76A)}$$

$$Q = 4.84 \text{ mL/s} \quad \text{(Eq. 76B)}$$

$$(L^3T^{-1}) \quad \text{(Eq. 76C)}$$

Common units:

$$Q = 4.84 \text{ mL/s} \times 10^{-3} \text{ L/mL} \times 60 \text{ s/min} \quad \text{(Eq. 77A)}$$

$$Q = 0.29 \text{ L/min} \quad \text{(Eq. 77B)}$$

Mass flow rate:

$$dM/dt = \rho\ Q \quad \text{(Eq. 78A)}$$

$$(MT^{-1} = ML^{-3} \times L^3T^{-1}) \quad \text{(Eq. 78B)}$$

$$dM/dt = 0.843 \text{ kg/L} \times (4.84 \times 10^{-3} \text{ L/s}) \quad \text{(Eq. 78C)}$$

$$dM/dt = 4.08 \times 10^{-3} \text{ kg/s} \quad \text{(Eq. 78D)}$$

$$dM/dt = 4.08 \text{ g/s} \quad \text{(Eq. 78E)}$$

Momentum:

$$v\ dM/dt = \rho\ Q\ v \quad \text{(Eq. 79A)}$$

$$LT^{-1} \times MT^{-1} = ML^{-3} \times L^{3}T^{-1} \times LT^{-1} \quad \text{(Eq. 79B)}$$

$$v\ dM/dt = 154\ m/s \times 4.08\ g/s \quad \text{(Eq. 79C)}$$

$$v\ dM/dt = 0.628\ N = 628\ mN \quad \text{(Eq. 79D)}$$

NOTE: This represents the force required to hold the nozzle and avoid its motion in the opposite direction of the fluid jet.

5.5 Pressure Momentum Equation

A free body in steady state flow is in balance under the combined effect of the momentum and the hydrostatic forces at the entry (1) and exit (2) sections:

$$(\rho\ Q\ v_2 - \rho\ Q\ v_1) + (A_1P_1 - A_2P_2) = \Sigma F \quad \text{(Eq. 80A)}$$

$$(MT^{-1} \times LT^{-1}) + (L^2 \times ML^{-1}T^{-2}) = (MLT^{-2}) \quad \text{(Eq. 80B)}$$

$$(MLT^{-2}) + (MLT^{-2}) = MLT^{-2}) \quad \text{(Eq. 80C)}$$

where:

A = area, (m^2) - (L^2)

P = pressure, (Pa) - ($ML^{-1}T^{-2}$)

Q = flow rate, (L/s) - (L^3T^{-1})

$\rho\ Q$ = mass flow rate, (kg/s) - (MT^{-1})

v = velocity, (m/s) - (LT^{-1})

ΣF = sum of forces, (N) - (MLT^{-2})

NOTES:

1. Equation 80A is known as the Euler equation.
2. In the example given in 5.4.5, this represents the force required to hold the nozzle and avoid its motion in the opposite direction of the fluid jet.
3. The force is acting in a single motion axis. The force is the external force to the fluid on this axis and is counted positively when acting in the opposite direction of the entry flow.

Practical Units

$$1\ mm^2 = 10^{-6}\ m^2 \quad \text{(Eq. 81A)}$$

$$1\ Mpa = 10^6\ N/m^2 \quad \text{(Eq. 81B)}$$

Therefore:

$$1\ mm^2 \times 1\ Mpa = 1\ N \quad \text{(Eq. 82A)}$$

$$1\ mm^2 \times 1000\ kPa = 1\ N \quad \text{(Eq. 82B)}$$

5.5.1 Application - Sudden Expansion, Borda-Carnot Equation

The jet from the 0.20 mm diameter nozzle of example 5.4.5 enters into a duct of 0.37 mm inside diameter. The flow expands to fill the larger section through turbulent mixing.

a. What is the steady-state velocity (V_2), the momentum (ρ Q v_2) and the pressure (P_2) at the expanded section? What is the loss in total pressure (ΔP)?

b. Check the result using the Borda-Carnot equation to determine the total pressure loss in a sudden expansion.

Neglecting elevation head and taking the entry pressure for datum, the pressure momentum equation becomes:

$$(\rho\ Q\ v_2 - \rho\ Q\ v_1) - (A_2 P_2) = 0 \qquad \text{(Eq. 83A)}$$

$$((ML^{-3} \times L^3T^{-1} \times LT^{-1}) - (L^2 \times ML^{-1}T^{-2}) = 0) \qquad \text{(Eq. 83B)}$$

where:

Subscript 1 refers to the entry and 2 refers to the exit - expanded pipe.

ρ Q v_1 = 628 mN jet momentum per 5.4.5

ρ Q v_2 = duct momentum, (mN) - (10^{-3}N, MLT^{-2})

A_2 = duct area, (mm^2) - (10^{-6} m^2, L^2)

P_2 = duct pressure, (kPa) - (10^3 Pa, ML^{-1} T^{-2})

The duct area is:

$$A = \pi D^2/4 \qquad \text{(Eq. 84A)}$$

$$A = \pi(0.37)^2/4 \qquad \text{(Eq. 84B)}$$

$$A = 0.107\,5\ mm^2 \qquad \text{(Eq. 84C)}$$

At the exit section (2), where flow expands to the entire duct area, velocity is:

$$v_2 = Q/A_2 \qquad \text{(Eq. 85)}$$

The flow is:

$$Q = 4.84\ mL/s \text{ per 5.4.5} \qquad \text{(Eq. 86)}$$

NOTE: The flow is assumed to be the same for illustrative purposes only as in reality it would not the same because of the change in downstream pressure.

Therefore:

$$v_2 = 4.84\ mL/s\ /\ (0.1075\ mm^2) \qquad \text{(Eq. 87A)}$$

$$v_2 = 45\ m/s \qquad \text{(Eq. 87B)}$$

Mass flow rate:

$$dM/dt = 4.08\ g/s \text{ per 5.4.5} \qquad \text{(Eq. 88)}$$

Momentum:

$$dM/dt\ v_2 = 4.08\ g/s \times 45\ m/s \quad (Eq.\ 89A)$$

$$dM/dt\ v_2 = 183\ mN \quad (Eq.\ 89B)$$

$$\rho\ Q\ v_2 = 183\ mN \quad (Eq.\ 89C)$$

The change of momentum is:

$$\rho\ Q\ v_2 - \rho\ Q\ v_1 = 628\ mN - 183\ mN \quad (Eq.\ 90A)$$

$$\rho\ Q\ v_2 - \rho\ Q\ v_1 = 445\ mN \quad (Eq.\ 90B)$$

The pressure forces on the duct equal the change of momentum:

$$A_2P_2 = \rho\ Q\ v_2 - \rho\ Q\ v_1 \quad (Eq.\ 91)$$

The pressure:

$$P_2 = (\rho\ Q\ v_2 - \rho\ Q\ v_1) / A_2 \quad (Eq.\ 92A)$$

$$P_2 = 445\ mN / 0.1075\ mm^2 \quad (Eq.\ 92B)$$

$$P_2 = 4140\ kPa \quad (Eq.\ 92C)$$

The total pressure of the jet is 10000 kPa at the duct inlet (5.4.5). As the jet diffuses and fills the duct through turbulent mixing, the total pressure becomes:

$$P_2 + \rho\ v_2^2/2 \quad (Eq.\ 93)$$

where:

ρ = 0.843 kg/L - ($10^3 kg/m^3$, ML^{-3})

v_2 = 45 m/s, mean velocity in the duct - (LT^{-1})

Velocity pressure:

$$\rho\ v_2^2/2 = 0.843\ kg/L \times (45\ m/s)^2/2 \quad (Eq.\ 94A)$$

$$\rho\ v_2^2/2 = 854\ kPa \quad (Eq.\ 94B)$$

Total pressure:

$$P_2 + \rho\ v_2^2/2 = 4140\ kPa + 854\ kPa \quad (Eq.\ 95A)$$

$$P_2 + \rho\ v_2^2/2 = 4994\ kPa \quad (Eq.\ 95B)$$

Drop in total pressure:

$$\Delta P = 10000\ kPa - 4994\ kPa \quad (Eq.\ 96A)$$

$$\Delta P = 5006\ kPa \quad (Eq.\ 96B)$$

The Borda-Carnot equation for total pressure loss in a sudden expansion is:

$$\Delta P = \rho\ (v_1 - v_2)^2 / 2 \quad \text{(Eq. 97A)}$$

$$(ML^{-1}T^{-2} = ML^{-3} \times (LT^{-1})^2) \quad \text{(Eq. 97B)}$$

where:

ρ = 0.843 kg/L - (10^3kg/m^3, ML^{-3})

v_1 = 154 m/s - (LT^{-1})

v_2 = 45 m/s - (LT^{-1})

Yielding:

$$\Delta P = 0.843 \text{ kg/L} \times (154 \text{ m/s to } 45 \text{ m/s})^2 / 2 \quad \text{(Eq. 98A)}$$

$$\Delta P = 5006 \text{ kPa, verifying the preceding results} \quad \text{(Eq. 98B)}$$

5.6 Bulk Modulus

5.6.1 Secant/Tangent and Adiabatic/Isothermal Bulk Modulus

The bulk modulus of compressibility of hydraulic fluids is defined with reference to the curve of the pressure change versus the relative volume variation.

The secant bulk modulus is the slope of the secant to the curve which is generally drawn through the origin or the mean operating point.

It is defined by the equation:

$$B_C = V_0\ (P_1 - P_0) / (V_0 - V_1) \quad \text{(Eq. 99A)}$$

$$(ML^{-1}T^{-2} = (L^3 \times ML^{-1}T^{-2}) / L^3) \quad \text{(Eq. 99B)}$$

where:

P_0 = initial pressure, usually zero gauge, (Pa) - ($ML^{-1}T^{-2}$)

P_1 = final pressure, (Pa) - ($ML^{-1}T^{-2}$)

ΔP = finite variation in pressure, (P) ($ML^{-1}T^{-2}$)

V_0 = initial fluid volume, (L) - (m^3, L^3)

V_1 = final fluid volume, (L) - (m^3, L^3)

ΔV = finite variation in volume, (L) - (m^3, L^3)

Let:

$$\Delta P = P_1 - P_0 \quad \text{(Eq. 100A)}$$

$$(ML^{-1}T^{-2}) \quad \text{(Eq. 100B)}$$

$$\Delta V = V_0 - V_1 \quad \text{(Eq.101A)}$$

$$(L^3) \quad \text{(Eq.101B)}$$

$$B_S = V_0(\Delta P) / (\Delta V) \quad \text{(Eq. 102)}$$

The tangent bulk modulus is the slope of the tangent to the curve at the pressure P of interest. It is defined by:

$$B_T = V_0(\delta P/\delta V) \quad \text{(Eq. 103A)}$$

$$(ML^{-1}T^{-2} = L^3 \times ML^{-1}T^{-2}/L^3) \quad \text{(Eq. 103B)}$$

where:

δP = infinitely small variation in pressure, (Pa) - ($ML^{-1}T^{-2}$)

δV = infinitely small variation in volume, (m^3) - (L^3)

The conditions usually considered for bulk modulus determination are the isothermal process (at constant temperature) or the adiabatic process (with no heat transfer).

The compression of a hydraulic fluid produces heat. In an isothermal process, the heat of compression is dissipated to the environment. In an adiabatic process, the heat of compression remains in the fluid. The internal heat results in thermal expansion of the fluid, which increases the resistance to the compression. As a result, the adiabatic bulk modulus of compressibility is larger than the isothermal bulk modulus.

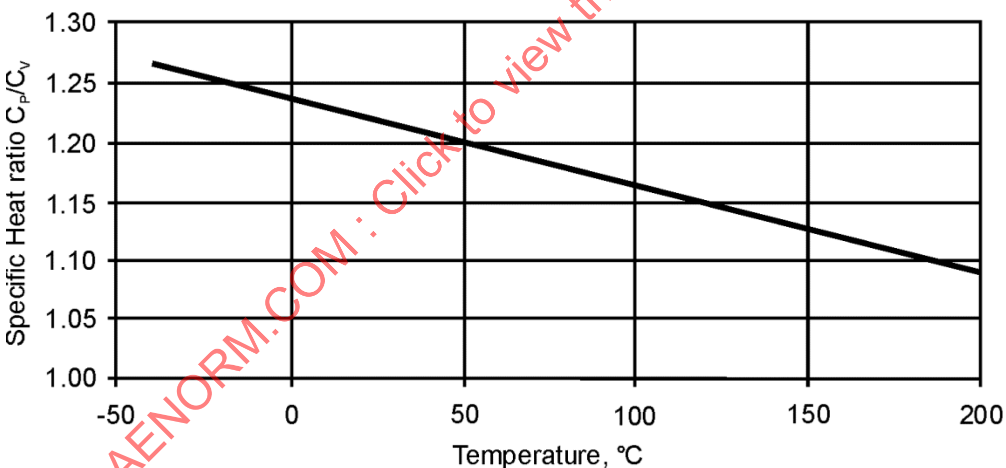

Figure 5 - Specific heat ratio of hydraulic fluids

The isothermal bulk modulus applies to processes that are sufficiently slow for removal of the heat of compression to the environment.

The adiabatic tangent bulk modulus at a given pressure level is used in the determination of:

a. The speed of sound in the hydraulic fluid

b. Water hammer

c. Organ pipe resonance

5.6.1.1 Speed of Sound

The speed of sound in the hydraulic fluid in a rigid tube with no free gas is:

$$C = (B_{AT}/\rho)^{0.5} \quad \text{(Eq. 104A)}$$

$$(LT^{-1} = (ML^{-1}T^{-2} / ML^{-3})^{0.5}) \quad \text{(Eq. 104B)}$$

where:

B_{AT} = adiabatic tangent bulk modulus, (Pa) - ($ML^{-1}T^{-2}$)

ρ = mass density, (kg/m^3) - (ML^{-3})

The adiabatic tangent bulk modulus (B_{AT}) is replaced by the effective bulk modulus (B_E) to account for the walls' deformation and the presence of free gas.

In determining the effective bulk modulus, dissolved gas has little or no effect on the bulk modulus of the hydraulic fluid. However, entrained gas has a significant effect, especially at low operating pressures, such as in the pump suction line. For example, 1% entrained gas by volume in the fluid reduces the effective bulk modulus to approximately 5% of the fluid without any entrained gas with a typical hydraulic reservoir pressure of 345 kPa (50 psi).

The mechanical compliance effect of the transmission line can usually be ignored for tubing and piping but must be considered for a flexible hose.

It is assumed that the effective bulk modulus of hose and fluid is in the 200000 to 400000 kPa (29000 to 58000 psi) range.

5.6.1.1.1 Numerical Example

Calculate the speed of sound of MIL-PRF-83282 at 55000 kPa pressure and 100 °C:

In primary units:

From Figure 3, ρ = 0.837 kg/L - (ML^{-3})

From Figure 6, B_{AT} = 1.87 x10^9 Pa - ($ML^{-1}T^{-2}$)

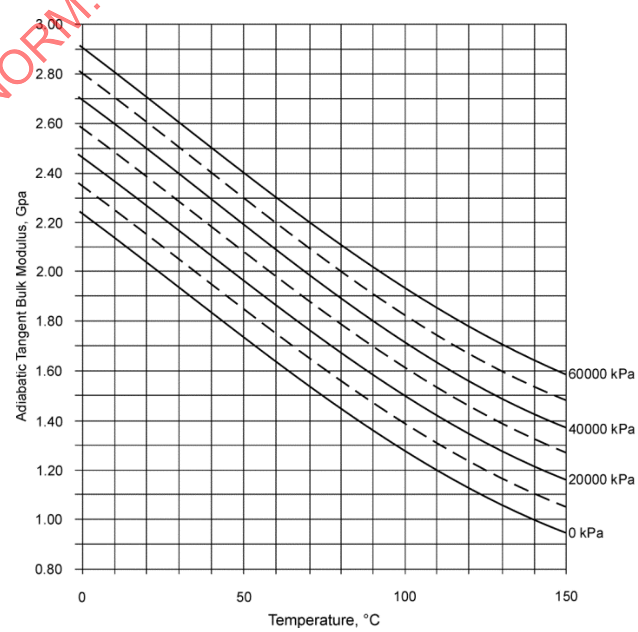

Figure 6 - Adiabatic tangent bulk modulus, MIL-PRF-83282

$$C = (1.87 \times 10^9/0.837)^{0.5} \quad \text{(Eq. 105A)}$$

$$(LT^{-1} = (ML^{-1}T^{-2} / ML^{-3})^{0.5}) \quad \text{(Eq. 105B)}$$

$$C = 10^3(1.87/0.837)^{0.5} \quad \text{(Eq. 105C)}$$

$$C = 1495 \text{ m/s} \quad \text{(Eq. 105D)}$$

NOTE: Where 1495 m/s is taken as the speed of sound, this is for a pure liquid (no gas), infinite rigid walls and with the effective bulk modulus of the fluid.

In practical units:

ρ = 0.837 kg/L - (10^3kg/m^3, ML^{-3})

B_{AT} = 1.87 Gpa - (10^9Pa, $ML^{-1}T^{-2}$)

$$C = (1.87 \text{ Gpa}/0.837 \text{ kg/L})^{0.5} \quad \text{(Eq. 106A)}$$

$$C = 1495 \text{ m/s} \quad \text{(Eq. 106B)}$$

5.6.1.2 Water Hammer Pressure Due to a Sudden Velocity Reduction of a Long Fluid Column in a Rigid Tube

$$\Delta P = \rho\ C\ \Delta v \quad \text{(Eq. 107A)}$$

$$(ML^{-1}T^{-2} = ML^{-3} \times LT^{-1} \times LT^{-1}) \quad \text{(Eq. 107B)}$$

5.6.1.2.1 Numerical Example

Calculate the water hammer pressure (ΔP) due to a sudden velocity reduction (Δv) of 5 m/s.

With the same conditions as 5.6.1.1.1:

ρ = 0.837 kg/L - (ML^{-3})

C = 1495 m/s - (LT^{-1}) from equation (Equation 106B)

Δv = 5 m/s - (LT^{-1})

$$\Delta P = 0.837 \times 1495 \times 5 \quad \text{(Eq. 108A)}$$

$$(M/L^2) = (ML^{-3}) \times (LT^{-1}) \quad \text{(Eq. 108B)}$$

$$\Delta P = 6.26 \times 10^3 \text{ Pa} \quad \text{(Eq. 108C)}$$

In practical units:

ρ = 0.837 kg/L - (10^3kg/m^3, ML^{-3})

C = 1495 m/s - (LT^{-1})

Δv = 5 m/s

$$\Delta P = 0.837 \times 1495 \times 5 \quad \text{(Eq. 109A)}$$

$$\Delta P = 6257 \text{ kPa} \quad \text{(Eq. 109B)}$$

5.6.1.3 Organ Pipe Resonance

Standing waves can be established in some hydraulic lines by pulsating pressures. Organ pipe resonance occurs when the length of a straight hydraulic line run equals one-quarter of the pressure or flow excitation wavelength as shown in Equation 110.

$$L = 0.25\ \lambda \quad \text{(Eq. 110)}$$

The wavelength is:

$$\lambda = C/f \quad \text{(Eq. 111A)}$$

$$(L = LT^{-1} / T^{-1}) \quad \text{(Eq. 111B)}$$

where:

f = frequency of the pulsations, (Hz) - (T^{-1})

5.6.1.4 Numerical example - For a given length of pipe at what pump speed does the organ pipe resonance appear?

The pipe length is 0.60 m.

The fundamental pulsation frequency of a 9 piston hydraulic pump speed as a function of rpm is:

$$f = (9 \times rpm)/60\ Hz \quad \text{(Eq. 112)}$$

C = 1495 m/s - (LT^{-1}) from equation (Equation 106B)

Then:

Substituting the values from Equation 111A

$$\lambda = (1495 \times 60)/(9 \times rpm) \quad \text{(Eq. 113A)}$$

$$(L = LT^{-1} / T^{-1}) \quad \text{(Eq. 113B)}$$

$$\lambda = (9967/rpm)\ m \quad \text{(Eq. 113C)}$$

L = 0.25 λ from Equation 110.

Substituting the pipe length and the value from Equation 113C.

$$0.6 = ((0.25 \times 9967)/rpm)\ m \quad \text{(Eq. 114A)}$$

$$rpm = (2492/0.6) \quad \text{(Eq. 114B)}$$

$$(T^{-1} = LT^{-1} / L) \quad \text{(Eq. 114C)}$$

$$\text{Pump speed} = 4153\ rpm \quad \text{(Eq. 114D)}$$

5.7 Adiabatic Secant Bulk Modulus Effects

5.7.1 Practical Units

Figure 7 shows how the Adiabatic Secant Bulk Modulus varies with temperature and pressure for a typical hydraulic fluid (in this case MIL-PRF-82382).

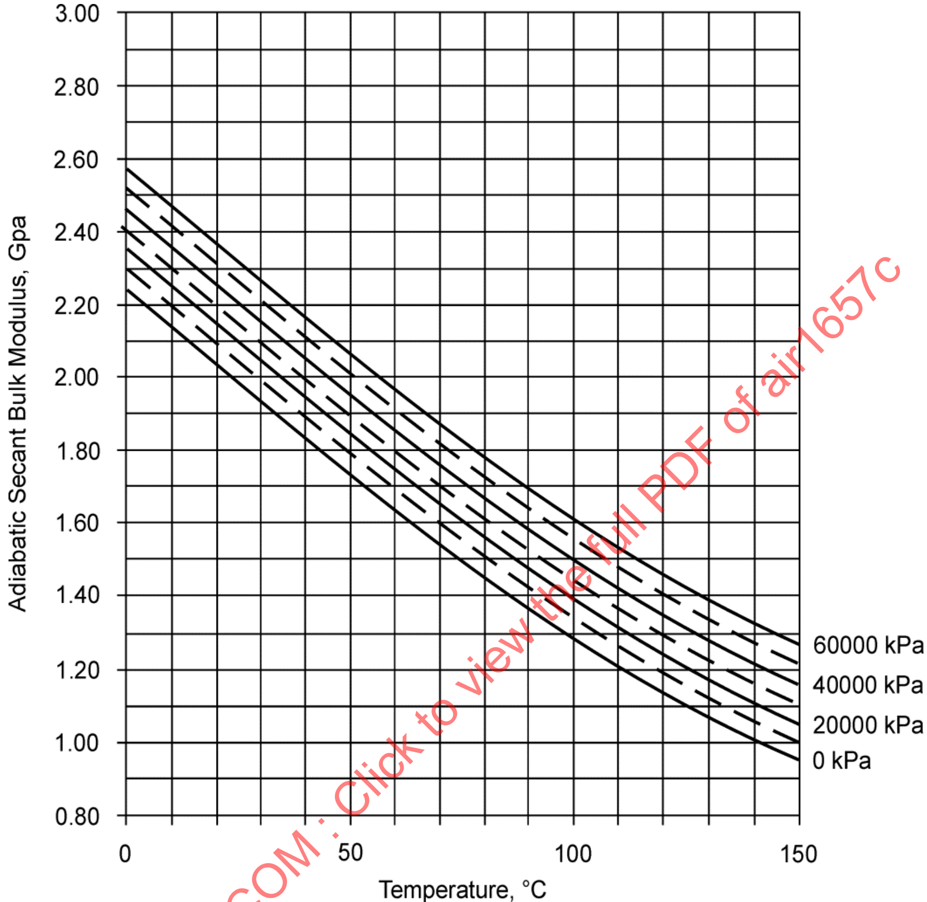

Figure 7 - Adiabatic secant bulk modulus, MIL-PRF-83282

5.7.2 Application - Pressure Rise Time

It is intended to establish a nominal system pressure (P) of 20000 kPa by running up a hydraulic pump at (N) 6000 rpm using MIL-PRF-83282 hydraulic fluid at 100 °C.

The pump displacement (V_p/rev) is 18.9 mL/revolution, and the volumetric efficiency (η) is 96%.

The system fluid volume (V) to be pressurized is 4 L.

Calculate pump discharge, the rate of system pressure rise (dP/dt) and the time (Δt) required to increase pressure to the nominal system pressure.

NOTE: For the purpose of this application, it is assumed that the hydraulic reservoir pressure has a negligible effect on the calculations.

The pump flow is:

$$Q = \eta \times (V_p/rev) \times N \quad \text{(Eq. 115A)}$$

$$(L^3 \times T^{-1}) \quad \text{(Eq. 115B)}$$

where:

$\eta = 0.96$

(V_p/rev) = 18.9 mL/rev - (L^3)

$$N = 6000 \text{ rpm} / (60 \text{ s/min}) \quad \text{(Eq. 116A)}$$

$$N = 100 \text{ rev/s } (T^{-1}) \quad \text{(Eq. 116B)}$$

NOTE: The SI system for rotational speed is rad/s, but the pump speed is universally referred to in terms of rev/minute (rpm).

Flow:

$$Q = 0.96 \times 18.9 \text{ mL/rev} \times 100 \text{ rev/s} \quad \text{(Eq. 117A)}$$

$$Q = 1\,810 \text{ mL/s} = 1.81 \text{ L/s} \quad \text{(Eq. 117B)}$$

$$(L^3T^{-1}) \quad \text{(Eq. 117C)}$$

The rate of pressure rise is given by:

$$dP/dt = B \times Q/V \quad \text{(Eq. 118A)}$$

$$(ML^{-1}T^{-3} = ML^{-1}T^{-2} \times L^3T^{-1}/L^3) \quad \text{(Eq. 118B)}$$

where

B = 1.4 Gpa - $(Pa^9, ML^{-1}T^{-2})$ - from Figure 7.

V = 4 L - (m^3, L^3)

Therefore:

$$dP/dt = 1.4 \text{ Gpa} \times 1.81 \text{ L/s} / 4L \quad \text{(Eq. 119A)}$$

$$dP/dt = 0.63 \text{ Gpa/s} \quad \text{(Eq. 119B)}$$

$$dP/dt = 630000 \text{ kPa/s} \quad \text{(Eq. 119C)}$$

The rise time is given by:

$$\Delta t = \Delta P / (dP/dT) \quad \text{(Eq. 120A)}$$

$$(T = ML^{-1}T^{-2} / (ML^{-1}T^{-3})) \quad \text{(Eq. 120B)}$$

where:

ΔP = 20000 kPa - 0 kPa = 20000 kPa (Pa^3, ML^{-1}T^{-2})

$$\Delta t = 20000/(630000 \text{ kPa/s}) \quad \text{(Eq. 121A)}$$

$$\Delta t = 0.032 \text{ s (T)} \quad \text{(Eq. 121B)}$$

The rise time is:

$$\Delta t = 32 \text{ ms} \quad \text{(Eq. 124)}$$

6. USE OF METRIC UNITS IN HYDRAULIC SYSTEM ANALYSIS - THERMODYNAMICS

6.1 Warm-Up of Hydraulic Fluid

6.1.1 Power and Heat Parameters

Table 5 contains details of the symbols, the physical quantities and the units that are used in power and heat calculations.

Table 5 - Symbols, physical quantities, and units used in power and heat calculations

Description	Symbol	Dimension	SI Metric Unit	Practical Metric Unit
Power consumption	W	ML^2T^{-3}	W	kW
Volumetric flow	Q_v	L^3T^{-1}	m^3/s	L/s
Pressure drop	ΔP	$ML^{-1}T^{-2}$	Pa	kPa
Heat flow	Q_h	ML^2T^{-3}	W	kW
Density	ρ	ML^{-3}	kg/m^3	kg/L
Specific heat	C_p	L^2T^{-2}	J/(kg·K)	kJ/ J/(kg·K)
Temperature rise	ΔT		K	°C
Elapsed warm-up time	t	T	s	s
Rate of temperature change	dT/dt	T^{-1}	K/s	°C/s
Efficiency	η			
Volume of hydraulic system	V	L^3	m^3	L

6.1.2 Power Conversion Equations

The hydraulic power (W) consumed by a flow Q_v under a pressure drop ΔP is:

$$W = Q_v \, \Delta P \quad \text{(Eq. 125A)}$$

$$(ML^2T^{-3} = L^3T^{-1} \times ML^{-1}T^{-2}) \quad \text{(Eq. 125B)}$$

NOTE: This assumes that hydraulic fluid is incompressible.

The heat flow (Q_h) of the fluid of density ρ and specific heat C_p at a temperature rise ΔT:

$$Q_h = \rho \, C_p \, Q_v \, \Delta T \quad \text{(Eq. 126A)}$$

$$(ML^2T^{-3} = ML^{-3} \times L^2T^{-2} \times L^3T^{-1}) \quad \text{(Eq. 126B)}$$

Equating the heat generated by throttling to the power consumed (assuming that all the heat generated goes into the fluid and none goes to the tubing):

$$\rho\ C_p\ Q_v\ \Delta T = Q_v\ \Delta P \quad \text{(Eq. 127A)}$$

$$(ML^{-3} \times L^2T^{-2} \times L^3T^{-1} = L^3T^{-1} \times ML^{-1}T^{-2}) \quad \text{(Eq. 127B)}$$

Cancelling the volume flow on each side of Equation 127A becomes:

$$\rho\ C_p\ Q_v\ \Delta T = Q_v\ \Delta P \quad \text{(Eq. 128A)}$$

$$\rho\ C_p\ \Delta T = \Delta P \quad \text{(Eq. 128B)}$$

The temperature rise across the flow restrictor is directly proportional to the pressure drop:

$$\Delta T = \Delta P\ /\ (\rho\ C_p) \quad \text{(Eq. 129A)}$$

$$(ML^{-1}T^{-2}\ /\ ML^{-3} \times L^2T^{-2}) \quad \text{(Eq. 129B)}$$

The number of passes, n, required in a closed loop system to increase the fluid temperature from an initial value of T_i to a final temperature of T_f:

$$n = (T_f - T_i)\ /\ \Delta T \quad \text{(Eq. 130)}$$

Duration of each pass:

$$\Delta t = V\ /\ Q_v \quad \text{(Eq. 131A)}$$

$$(T = L^3\ /\ L^3T^{-1}) \quad \text{(Eq. 131B)}$$

Time required for the warm-up without heat transfer to the environment:

$$\tau = n\ \Delta t \quad \text{(Eq. 132A)}$$

$$(T = T) \quad \text{(Eq. 132B)}$$

where:

τ = time constant, (s) - (T)

n = number of passes

Δt = duration of each pass, (s) - (T)

As the hydraulic fluid temperature rises above the ambient, heat will flow to the environment. The heat loss slows down the temperature rise. A thermal equilibrium temperature (ΔT_f) is approached asymptotically. The temperature rise ΔT_x versus the elapsed time t_x follows approximately the exponential law:

$$\Delta T_x = \Delta T_f\ (1 - \exp\ (-t_x\ /\ \tau)) \quad \text{(Eq. 133)}$$

where:

ΔT_f = final equilibrium temperature rise

τ = the time constant determined in Equations 131 and 132 provided that the ambient temperature is the same as the initial fluid temperature T_i, and T_f is taken as the final thermal equilibrium temperature.

6.1.3 Application - Warm-Up by Throttling

MIL-PRF-83282 hydraulic fluid is warmed up from a -20 °C cold start to a final thermal equilibrium temperature of 40 °C. The ambient temperature stays at -20 °C.

The fluid flow is 120 L/min.

The pressure drop through the throttle valve is 17000 kPa.

The fluid volume in the hydraulic system is 100 L.

Calculate:

a. The hydraulic power consumption

b. The temperature rise across the throttle valve

c. The number of passes required assuming no heat losses

d. The duration of one pass

e. The length of time required for the warm-up at the initial rate

f. The percentage of the equilibrium temperature achieved in 15 min

g. The temperature at the end of 15 minutes

The MIL-PRF-83282 hydraulic fluid properties are:

At -20 °C

ρ = 0.88 kg/L, fluid density, Figure 3

C_p = 1.87 kJ/kg K, fluid specific heat, Figure 8

At 40 °C

ρ = 0.84 kg/L, fluid density, Figure 3

C_p = 2.09 kJ/kg K, fluid specific heat, Figure 8

Use mean values between -20 and 40 °C

ρ = 0.86 kg/L, fluid density, Figure 3

C_p = 1.98 kJ/kg K, fluid specific heat, Figure 8

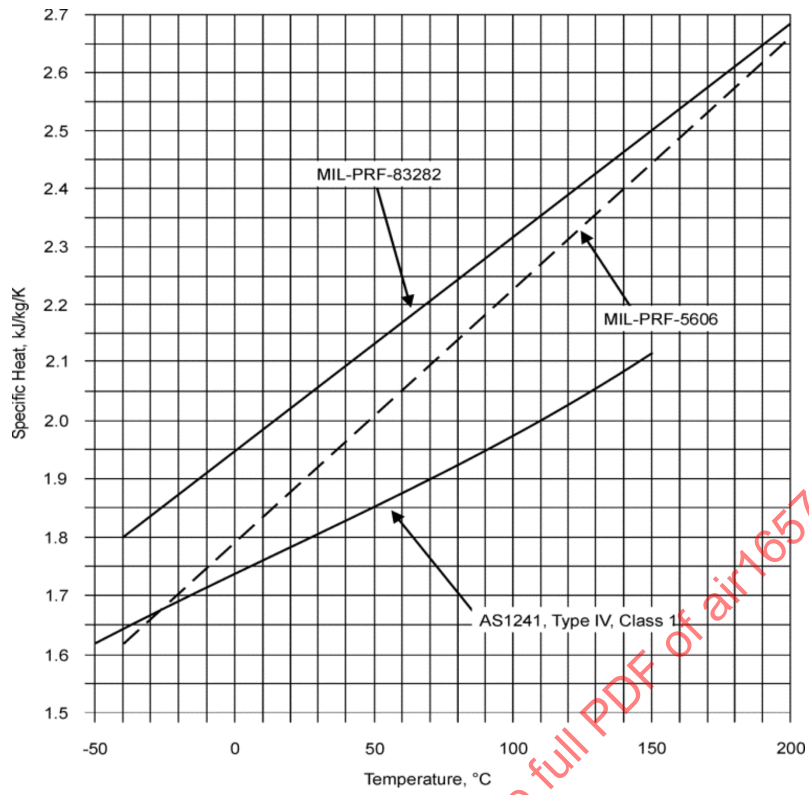

Figure 8 - Specific heat of hydraulic fluids

Unit conversion:

$$120 \text{ L/min} = (120 \text{ L/min}) / (60 \text{ s/min}) \quad \text{(Eq. 134A)}$$

$$120 \text{ L/min} = 2.0 \text{ L/s} \quad \text{(Eq. 135B)}$$

a. The power consumption is by Equation 125:

$$W = (2.0 \text{ L/s} \times 17000 \text{ kPa})/1000 \quad \text{(Eq. 136A)}$$

$$W = 34 \text{ kW} \quad \text{(Eq. 136B)}$$

b. The temperature rise is by Equation 129:

$$\Delta T = 17000 \text{ kPa} / (1000 \times 0.86 \text{ kg/L} \times 1.98 \text{ kJ/kg K}) \quad \text{(Eq. 137A)}$$

$$\Delta T = 10\ °C \quad \text{(Eq. 137B)}$$

c. The duration of one pass is by Equation 131:

$$\Delta t = 100 \text{ L} / (2.0 \text{ L/s}) \quad \text{(Eq. 138A)}$$

$$\Delta t = 50 \text{ s each pass} \quad \text{(Eq. 138B)}$$

d. The final temperature rise to the equilibrium temperature is obtained as:

$$\Delta T_f = 40\ °C - (-20\ °C) \quad \text{(Eq. 139A)}$$

$$\Delta T_f = 60\ °C \quad \text{(Eq. 139B)}$$

e. The number of passes required without heat loss is given by Equation 130:

$$n = 60\ °C / 10\ °C \quad \text{(Eq. 140A)}$$

$$n = 6 \text{ passes} \quad \text{(Eq. 140B)}$$

f. The length of time required for the warm-up at the initial rate by Equation 132:

$$\tau = 6 \times 50 \text{ s} \quad \text{(Eq. 141A)}$$

$$\tau = 300 \text{ s} \quad \text{(Eq. 141B)}$$

Conversion:

$$\tau = 300 \text{ s} / (60 \text{ s/min}) \quad \text{(Eq. 142A)}$$

$$\tau = 5\text{min} \quad \text{(Eq. 142B)}$$

Where τ is the time required for warm-up and also defines the warm-up time constant of the hydraulic system:

g. The percentage of the final temperature rise achieved in 15 min with heat loss to the environment is given by Equation 133:

$$\Delta T_x / \Delta T_f = (1 - \exp(-t_x/\tau)) \quad \text{(Eq. 143)}$$

where:

t_x = 15 min

τ = 5 min

Calculate:

$$(1 - \exp(-t_x/\tau)) = 1 - \exp(-15/5) \quad \text{(Eq. 144A)}$$

$$\Delta T_x / \Delta T_f = 95\ \% \quad \text{(Eq. 144B)}$$

The final temperature rise is:

$$\Delta T_x = 0.95 \times 60\ °C \quad \text{(Eq. 145A)}$$

$$\Delta T_x = 57\ °C \quad \text{(Eq. 145B)}$$

h. The temperature at the end of 15 min is 37 °C.

6.2 Cooling of Hydraulic Fluid

6.2.1 Heat Transfer Parameters

Table 6 contains details of the symbols, the physical quantities, and the units that are used in the power and heat calculations.

Table 6 - Symbols, physical quantities, and units used in heat transfer calculations

Name	Symbol	Dimensions	SI Metric Unit	Practical Metric Unit
Specific heat	C_p	L^2T^{-2}	J/kg K	kJ/kg K
Surface conductance	h_c	$ML^{-1}T^{-3}$	W/(m^2 K)	kW/m^2 K
Thermal conductivity	k	MLT^{-3}	W/(m K)	w/m K
Absolute (dynamic) viscosity	μ	$ML^{-1}T^{-1}$	Pa s	mPa s, centipoise
Kinematic viscosity	ν	L^2T^{-1}	m^2/s	mm^2/s, centistoke
Density	ρ	ML^{-3}	kg/m^3	kg/L
Length	l	L	m, meter	mm
Time	t	T	s, second	minute
Absolute Temperature	T	-	K, Kelvin	
Temperature	T	-	°C, degree Celsius (centigrade)	
Differential Temperature	ΔT_d	-	°C, degree Celsius	
Fluid Temperature Drop	ΔT_f	-	°C, degree Celsius	
Area	A	L^2	m^2	mm^2
Volume	V	L^3	m^3	L, liter, mm^3, μL
Velocity	v	LT^{-1}	m/s	mm/s
Flow, Volumetric	Q_v	L^3T^{-1}	m^3/s	L/s
Heat Flow	Q_h	ML^2T^{-3}	W, watt	kW
Heat Capacity Rate	c	ML^2T^{-3}	w/K	kW/K
Reynolds Number	Re			
Prandtl Number	Pr			
Nusselt Number	Nu			

6.2.2 Heat Transfer Equations

All basic equations are in primary units. Practical units are useful to facilitate calculations. See the third column in Table 6.

a. Kinematic viscosity, definition:

$$\nu = \mu / \rho \qquad \text{(Eq. 146)}$$

b. Absolute viscosity by solving Equation 133:

$$\mu = \nu \rho \qquad \text{(Eq. 147)}$$

c. Reynolds number, by definition:

$$Re = v D / \nu \qquad \text{(Eq. 148A)}$$

$$(LT^{-1} \times L / L^2T^{-1}) \qquad \text{(Eq. 148B)}$$

d. Prandtl number, by definition:

$$Pr = C_p \mu / k \qquad \text{(Eq. 149A)}$$

$$(L^2T^{-2} \times ML^{-1}T^{-1} / MLT^{-3}) \qquad \text{(Eq. 149B)}$$

e. Nusselt number, by definition:

$$Nu = h_c\,D / k \qquad \text{(Eq. 150A)}$$

$$(MT^{-3} \times L / MLT^{-3}) \qquad \text{(Eq. 150B)}$$

f. Nusselt number in turbulent flow:

$$Nu = 0.023\,Re^{0.8}\,Pr^{0.33} \qquad \text{(Eq. 151)}$$

NOTE: This equation applies to fully developed turbulent flow in smooth tubes if the Prandtl number > 0.7.

g. Surface conductance, by solving Equation 150:

$$h_c = Nu\,k / D \qquad \text{(Eq. 152A)}$$

$$(MT^{-3} = MLT^{-3} / L) \qquad \text{(Eq. 152B)}$$

h. Heat transfer area of the tube:

$$A = \pi\,D\,l \qquad \text{(Eq. 153)}$$

i. Heat flow through the surface area:

$$Q_h = h_c\,A\,\Delta T_d$$

$$(ML^2T^{-3} = MT^{-3} \times L^2) \qquad \text{(Eq. 154)}$$

j. Heat flow removed from the hot fluid:

$$Q_h = c\,\Delta T_f \qquad \text{(Eq. 155A)}$$

$$(ML^2T^{-3}) \qquad \text{(Eq. 155B)}$$

k. Cross sectional flow area of the tube:

$$S = \pi\,D^2/4 \qquad \text{(Eq. 156A)}$$

$$(L^2 = L^2) \qquad \text{(Eq. 156B)}$$

l. Volume flow:

$$Q_v = v\,S \qquad \text{(Eq. 157A)}$$

$$(L^3T^{-1} = LT^{-1} \times L^2) \qquad \text{(Eq. 157B)}$$

m. Heat capacity rate:

$$c = C_p\,\rho\,Q_v \qquad \text{(Eq. 158A)}$$

$$(ML^2T^{-3} = L^2T^{-2} \times ML^{-3} \times L^3T^{-1}) \qquad \text{(Eq. 158B)}$$

n. The fluid temperature drop:

$$\Delta T_f = Q_h / c \qquad \text{(Eq. 159A)}$$

$$(ML^2T^{-3} / ML^2T^{-3}) \qquad \text{(Eq. 159B)}$$